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GRANT NAG3-655
APRIL 1988

(NASA-CR-4135) SPUR GEARS: OPTIMAL GEOMETRY, METHODS FOR GENERATION AND TOOTH CONTACT ANALYSIS (TCA) PROGRAM Final Report (Illinois Univ.) 135 p CSCL 13I

N88-23216

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Spur Gears: Optimal Geometry, Methods for Generation and Tooth Contact Analysis (TCA) Program

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Prepared for
Propulsion Directorate
USAARTA-AVSCOM and
NASA Lewis Research Center
under Grant NAG3-655

National Aeronautics
and Space Administration

Scientific and Technical
Information Division

1988

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Abstract

The contents of this report covers: (i) development of optimal geometry for crowned spur gears; (ii) methods for their generation; and (iii) tooth contact analysis (TCA) computer programs for the analysis of meshing and bearing contact of the crowned spur gears.

The developed method for synthesis is used for the determination of the optimal geometry for crowned pinion surface and is directed to reduce the sensitivity of the gears to misalignment, localize the bearing contact, and guarantee the favorable shape and low level of the transmission errors.

A new method for the generation of the crowned pinion surface has been proposed. This method is based on application of the tool with a surface of revolution that slightly deviates from a regular cone surface. The tool can be used as a grinding wheel or as a shaver. The crowned pinion surface can be also generated by a generating plane whose motion is provided by an automatic grinding machine controlled by a computer.

The TCA program simulates the meshing and bearing contact of the misaligned gears. The transmission errors are also determined.

Summary

Spur gears are widely used to transmit mechanical power between parallel axes of drive shafts. Namely, such gears are used in planetary trains of helicopters. It is well known that spur involute gears are very sensitive to gear misalignments. Misalignment will cause: the shift of the bearing contact toward the edge of the gear tooth surfaces and transmission errors that increase gear noise. The previous methods for crowning of pinion tooth surface have been directed at providing of a favorable bearing contact. The transmission errors of the misaligned gears have been ignored. The new approach developed by the authors of this report is directed at the synthesis of spur involute gears with a crowned pinion surface that provides: (i) a parabolic type of low transmission errors for the misaligned gears and (ii) a localized bearing contact. New methods for the generation of crowned pinion surface proposed by the authors can be readily applied by the gear industry.

List of Symbols

a^*	half the length of major axis of contact ellipse
a_1	constant
a_i	auxiliary vector in coordinate system i
b	dedendum of cutting tool
b_1	constant
b^*	half the length of minor axis of contact ellipse
$[b_{ij}]$	3x3 auxiliary matrix
$\underset{\sim}{C}$	Operating center distance
C	operating center distance vector
$C^0 = \frac{N_1}{2P} + \frac{N_2}{2P}$	nominal center distance
$\underset{\sim}{C}^0$	nominal center distance vector
d	height of generating cone
$e_I^{(i)}$, $e_{II}^{(i)}$	unit vectors along principal direction of surface Σ_i if principal directions of two surface coincide, i is neglected
e_1	constant
$f_i(X_1,X_2...)$	function expression with respect to variable $X_1,X_2...$
$F_1(X_1,X_2...)$	function expression with respect to variable X_1, X_2
$F_i = \kappa_I^{(i)} - \kappa_{II}^{(i)}$	auxiliary function
$g(\phi_1)$	auxiliary function of ϕ_1
i_t, j_t, k_t	unit direction vectors of coordinate system t
K_i	auxiliary function
L_{ij}	line of tangency of surfaces Σ_i and Σ_j
$[L_{ij}]$	projection transformation matrix (from S_j to S_i)

m_i	auxiliary function
$m_{1p} = \frac{d\phi_p}{ds}$	kinematic ratio
$m'_{1p} = \frac{dm_{1P}}{ds}$	derivative of kinematic ratio of gear and rack cutter
$m_{21} = \frac{d\phi_2}{d\phi_1}$	kinematic ratio of gears
$m'_{21} = \frac{dm_{21}}{d\phi_1}$	derivative of kinematic ratio of gears
M_i	point i
$[M_{ij}]$	coordinate transformation matrix (from S_j to S_i)
$\dot{n}_r$	velocity of end point of unit normal corresponding a point moving over a surface
$n_i^{(j)}$	unit normal vector of surface Σ_i in coordinate system S_j [sometimes i or (j) is omitted if it is unnecessary to specify coordinate system]
N_1	number of pinion teeth
N2	number of gear teeth
O_i	origin of coordinate system i
p	auxiliary function
P	diametral pitch
q	auxiliary function
r_1	radius of pitch circle for pinion
r_2	radius of pitch circle for gear
R_1	radius of rotation to generate pinion tooth surface from a planar shape
$r_i^{(j)}(u,\theta)$	position vector describing surface Σ_j with surface coordinate (u,θ) in coordinate system S_i [sometimes (j) is omitted]
S	parameter of cutting motion
$S_i(X_i, Y_i, Z_i)$	coordinate system i

U_i	generating surface
U_p^*	auxiliary variable
$\underset{\sim}{V}_r$	velocity of a point moving over a surface
$\underset{\sim}{V}_j^{(i)}$	velocity of surface Σ_i in coordinate system j (j is omitted sometimes)
$V_f^{(ij)} = \underset{\sim}{V}_f^{(i)} - V_f^{(j)}$	relative velocity between surface Σ_i and Σ_j at contact point in coordinate system f (f is omitted sometimes).
$V_I^{(ij)}$, $V_{II}^{(ij)}$	projection of $V^{(ij)}$ in the principal direction of surface
$X_i^{(j)}$, $Y_i^{(j)}$, $Z_i^{(j)}$	coordinates of vector in S_i [sometimes (j) is omitted]
α	half of cone angle
β	generating surface coordinate
$\Delta\gamma$	crossing angle between axes of pinion and gear
$\Delta\delta$	intersecting angle between axes of pinion and gear
$\Delta\phi_2$	kinematic error
$\Delta\phi_2'$	derivative of kinematic error with respect to ϕ
ε	elastic deformation of the contacting point
θ_i	generating surface coordinate
$\kappa_I^{(i)}$, $\kappa_{II}^{(i)}$	principal curvatures of surface Σ_i
$\kappa_\varepsilon^{(i)} = \kappa_I^{(i)} + \kappa_{II}^{(i)}$	auxiliary function
λ	generating surface coordinate
ρ	radius of genetrix arc for revolute surface
$\sigma^{(ij)}$	angle between $e_I^{(i)}$ and $e_I^{(j)}$
Σ_1	pinion tooth surface
Σ_2	gear tooth surface (or shape)
Σ_G	gear generating surface

Σ_p — pinion generating surface

ϕ_p — angle of rotation for pinion being generated

ϕ_G — angle of rotation for gear being generated

ϕ_1 — angle of pinion rotation in meshing with gear

ϕ_2 — angle of gear rotation in meshing with pinion

ϕ_{20} — theoretical angle of gear rotation in meshing with pinion

ψ_c — pressure angle

$\omega^{(1)}$ — magnitude of $\tilde{\omega}^{(1)}$

$\omega^{(2)}$ — magnitude of $\tilde{\omega}^{(2)}$

$\tilde{\omega}$ — angular velocity of pinion being generated

$\tilde{\omega}^{(2)}$ — angular velocity of gear in meshing with $\tilde{\omega}^{(12)} = \tilde{\omega}^{(1)} - \tilde{\omega}^{(2)}$

$\tilde{\omega}^{(12)} = \tilde{\omega}^{(1)} - \tilde{\omega}^{(2)}$ — relative angular velocity

1. Introduction

Spur involute gears are very sensitive to gear misalignments that cause: (i) the shift of the bearing contact to the edge of the gear tooth surfaces and (ii) transmission errors that increase gear noise. Many efforts have been done to improve the bearing contact of misaligned spur gears by crowning of pinion tooth surface. Wildhaber (1962) has proposed various methods of crowning that can be achieved in the process of gear generation. Maag engineers (see References) have used crowning as (i) longitudinal corrections (Fig. 1.1a), (ii) modified involute tooth profile uniform across facewidth (Fig. 1.1b), (iii) combination of longitudinal correction and uniform modified profile (Fig. 1.1c) and (iv) topological modification (Fig. 1.1d) that can provide any deviation of the crowned tooth surface from a regular involute surface.

The main purpose of the previously proposed methods for crowning was to improve the bearing contact of the misaligned gears. This resulted in only part of the solution. The transmission errors of the misaligned spur gears (a main source of the noise), were ignored. The influence of gear misalignment on the transmission errors has not been investigated. Maag's method of topological modification does not quantify the relationship between the surface deviation and the transmission error. Also the optimal geometry for the crowned pinion surface has not been proposed.

The contents of this report cover the solutions to the following problems: (i) optimal geometry of a crowned pinion tooth surface; (ii) new method of crowning based on application of a tool with a surface of revolution (the tool surface is slightly deviated from a cone surface) (iii) the development of Tooth Contact Analysis (TCA) programs for the determination of transmission errors for misaligned gears and their bearing contact.

The development of the optimal geometry of a crowned pinion gear tooth surface is based on the following considerations:

(i) Misaligned spur gears with a crowned pinion tooth surface can provide transmission errors $\Delta\phi_2(\phi_1)$ of two types that are shown in Fig. 1.2a and 1.2b, respectively. The transmission errors are determined with the equation as

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\,\phi_1 \tag{1.1}$$

Here: N_1 and N_2 are the numbers of gear teeth; ϕ_1 and ϕ_2 are the angles of gear rotation; $\phi_2(\phi_1)$ is the function that relates the angles of rotation of gear and pinion if the pinion is crowned and the gears are misaligned; $\phi_{20} = \frac{N_1}{N_2}\,\phi_1$ is the theoretical relation between the angles of rotation of the gears in the ideal case where the gears are not crowned, not misaligned and the transmission errors do not exist. Type 1 of transmission errors are not acceptable because the change of tooth meshing is accompanied with an interruption or interference of tooth surfaces. Type 2 of transmission errors is more preferable if the level of the transmission errors does not exceed a prescribed limit.

(ii) On the first glance crowning should be directed at providing an exact involute shape in the middle cross-section (Fig. 1.3). In reality this type of crowning is not acceptable because the misaligned gears will transform rotation with transmission errors of type 1 (Fig. 1.2a). This is the reason why the authors have decided to synthesize a specific crowned pinion tooth surface. Such a pinion can provide transformation of rotation with a parabolic type of transmission error function when it is in meshing with a gear with a regular involute tooth surface. This type of function of transmission errors is synthesized for ideal gears that do not have any misalignment. Then, the tendency to provide parabolic transmission errors can be extended for the misaligned gears and the discontinuance of meshing can be avoided. It is necessary to emphasize that the proposed method of synthesis provides a shape in the middle cross-section of the pinion tooth that deviates in a certain way from the involute curve that is shown in Fig. 1.3. The longitudinal deviation from a straight line is not related with the transmission errors but with the desired dimensions of the instantaneous contact ellipse for the gear tooth surfaces. The proposed pinion tooth surface can be generated by a plane as the chosen generating surface. The motions of the plane with the respect to pinion must be controlled by a computer.

(iii) Another method of pinion crowning is based on application of a surface of revolution that slightly deviates from a regular tool conical surface (Fig. 1.4). Such a tool can be used as a grinding wheel or as a shaver. The motions of the tool and the gear being generated are related similarly to the motions of a rack-cutter and the gear (see Chapter 4). A tool with a regular conical surface can generate a crowned pinion tooth surface whose middle cross-section represents an involute curve (Fig. 1.3). However, this type of a crowned pinion tooth surface is not desirable because the misaligned gears provide transmission errors of type 1 (Fig. 1.2a). This is the reason why a tool with the surface of revolution is to be used instead of a conical surface.

(iv) The evaluation of the bearing contact and transmission errors for the misaligned gears as well as the investigation of the influence of errors of gear assembly requires application of TCA programs. Such programs have been developed by the authors. The programs are based on the following algorithms:

(a) The contacting gear tooth surfaces are represented in a fixed coordinate system, S_f, that is rigidly connected to the gear housing (Fig. 1.5).

(b) The continuous tangency of gear tooth surfaces is provided if the position-vectors and surface unit normals for the contacting surfaces coincide at the contact point at any instant. Then, we are able to determine the path of contact on the gear tooth surfaces and the relations between the angles of rotation of the output and input gears. Knowing function $\phi_2(\phi_1)$, we can determine the deviations of this function from the prescribed linear function i.e. the transmission errors.

(c) Due to the elasticity of the gear tooth surfaces the surface contact is spread over an elliptical area. The dimensions and orientation of the instantaneous contact ellipse depend on the principal curvatures and principal directions of the contacting tooth surfaces. The bearing contact is determined by the developed TCA program as the set of the contact ellipses that move over the contacting surfaces in the process of motion.

2. Optimal Pinion Geometry

2.1 Basic Consideration

It is assumed that only the pinion will be provided with a crowned tooth surface while the gear is provided with a regular involute surface. The crowned pinion tooth surface deviates from a regular involute surface. The topology of the crowned pinion tooth surface must satisfy the following requirements: (i) the pinion tooth surface is a continuous one; (ii) the pinion and gear tooth surfaces are in contact at a point at any instant; (iii) the principal directions and curvatures of the contacting surfaces must be related in such a way that the instantaneous contact ellipse will be of appropriate dimensions; (iv) the function of transmission errors of the misaligned gears should be of a parabolic type and its level must not extend the prescribed limits. The satisfication to the described conditions is considered in the following sections of this part.

2.2 Parabolic Type of Function of Kinematic Errors

Initially we will consider that the gears are not misaligned, the pinion tooth surface is crowned and the gear tooth surface is a regular involute surface. The tooth surfaces are in point contact at every instant and the points of contact lie in the middle cross-section. Thus, initially we consider the meshing of gear tooth surfaces as a planar gearing. However, when simulating the meshing of gears by TCA program, we will consider the mesh of them as spatial gears.

Figure 2.2.1 shows the shapes of the gears which are in contact at a point that lies on the center distance. The shape of the gear tooth profile is a regular involute curve, and the shape of the pinion tooth profile is to be determined. It is given that the shapes must provide transformation of rotation represented by the function (Fig. 2.2.2a)

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 + \Delta\phi_2(\phi_1) \tag{2.2.1}$$

where $\Delta\phi_2(\phi_1)$ is a function of a parabolic type that represents the function of transmission errors; N_1 and N_2 are the numbers of gear teeth. We have to emphasize that the proposed method for synthesis provides low transmission errors for aligned gears and the function of transmission errors is a parabolic one. Then, the misaligned gears will keep the tendency to transform rotation with the same type of transmission errors and the discontinuance of contact by meshing will be avoided. This means that we have to avoid the appearance of transmission errors that are shown in Fig. 1.2a. Function $\phi_2(\phi_1)$ contains a linear function represented by $\frac{N_1}{N_2}\phi_1$ that relates the angles of rotation of the "ideal" input and output gears. Figure 2.2.2a shows the function $\phi_2(\phi_1)$. The function of transmission errors is represented to a larger scale in Fig. 2.2.2b as a periodic function of a parabolic type. The derivative

$$\Delta\phi_2'(\phi_1)) = \frac{d}{d\phi_1}(\Delta\phi_2(\phi_1)) \tag{2.2.2}$$

is shown in Fig. 2.2.2c. For a "pure" parabolic function this derivative is a linear function. It is evident that the second derivative represented by

$$\frac{d^2}{d\phi_1^2}(\Delta\phi_2(\phi_1)) = \frac{d^2}{d\phi_1^2}(\phi_2(\phi_1))$$

is negative. The transformation of rotation by the gears will be performed with a non-constant gear ratio

$$m_{21} = \frac{\omega_2}{\omega_1} = \frac{N_1}{N_2} + \Delta\phi_2'(\phi_1) \tag{2.2.3}$$

The instantaneous center of rotation I is moved along the gear center distance in the process of gear rotation. Figure 2.2.1 shows the instantaneous location of I.

More details about the geometry of the pinion that corresponds to the discussed function $\phi_2(\phi_1)$ will be given in Chapter 3.

2.3 Relation Between the Principal Curvatures and Directions

The relation between the principal curvatures and directions of the contacting surfaces and the derivative of the gear ratio m_{21}' is represented as follows (Litvin, 1987)

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 0 \tag{2.3.1}$$

For the case of spur gears with the crowned pinion tooth surface the elements of the determinant of Eq. (2.3.1) are represented by the following equations

$$b_{21} = b_{12} \qquad b_{31} = b_{13} \qquad b_{32} = b_{23} \tag{2.3.2}$$

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} -\kappa_I^{(1)} + \kappa_I^{(2)} & 0 \\ 0 & -\kappa_{II}^{(1)} + \kappa_{II}^{(2)} \end{bmatrix} \tag{2.3.3}$$

$$\begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} = \begin{bmatrix} -\underset{\sim}{e}_I \cdot \underset{\sim}{\omega}^{(12)} \\ \underset{\sim}{e}_{II} \cdot \underset{\sim}{\omega}^{(12)} \end{bmatrix} \tag{2.3.4}$$

$$b_{33} = -\underset{\sim}{n} \cdot [(\underset{\sim}{\omega}^{(1)} x \underset{\sim}{V}_{tr}^{(2)}) - (\underset{\sim}{\omega}^{(2)} x \underset{\sim}{V}_{tr}^{(1)})] + (\omega^{(1)})^2 m_{21}' (\underset{\sim}{n} x \underset{\sim}{k}_2) \cdot (\underset{\sim}{r}_f^{(1)} - \underset{\sim}{C}) \tag{2.3.5}$$

Figure 2.3.1 shows the tangent plane to the gear tooth surfaces at the point of contact I (see Fig. 2.2.1). The principal directions are represented by the unit vectors $\underset{\sim}{e}_I$ and $\underset{\sim}{e}_{II}$ that are the same for the both contacting surfaces. Vector $\underset{\sim}{e}_I$ is shown in Fig. 2.3.1 and is represented in coordinate system S_f as follows

$$\underset{\sim}{e}_I = \begin{bmatrix} \sin\psi_c \\ -\cos\psi_c \\ 0 \end{bmatrix} \tag{2.3.6}$$

where ψ_c is the pressure angle at point I. Vector $\underset{\sim}{e}_{II}$ is directed parallel to the negative z_f-axis and is represented by

$$\underset{\sim}{e}_{II} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad (2.3.7)$$

The surface unit normal vector is represented by (Fig. 2.2.1)

$$\underset{\sim}{n}_f = \underset{\sim}{e}_I \times \underset{\sim}{e}_{II} = \begin{bmatrix} \cos\psi_c \\ \sin\psi_c \\ 0 \end{bmatrix} \quad (2.3.8)$$

The angular velocities of rotation are represented by (Fig. 2.2.1)

$$\underset{\sim}{\omega}^{(1)} = \omega^{(1)} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \qquad \underset{\sim}{\omega}^{(2)} = \omega^{(2)} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (2.3.9)$$

Vectors $\underset{\sim}{V}_{tr}^{(1)}$ and $\underset{\sim}{V}_{tr}^{(2)}$ represent the velocities of the contact point in transfer motion, while gears rotate. Thus (Fig. 2.2.1)

$$\underset{\sim}{V}_{tr}^{(1)} = \omega^{(1)}\, O_1I \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \underset{\sim}{V}_{tr}^{(2)} = \omega^{(2)} O_2I \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (2.3.10)$$

Vector $\underset{\sim}{C}$ and $\underset{\sim}{r}_f^{(1)}$ are

$$\underset{\sim}{C} = O_1O_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \underset{\sim}{r}^{(1)} = O_1I \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (2.3.11)$$

where $O_1O_2 = C^0 = \frac{N_1}{2p} + \frac{N_2}{2p}$, $\quad O_1I = \frac{N_1}{2p} \quad O_2I = \frac{N_2}{2p}$; p is diametral pitch and C° is ideal or nominal central distance.

$\kappa_I^{(2)}$ and $\kappa_{II}^{(2)}$ are the first and the second principal curvatures of the gear tooth surface represented by

$$\kappa_I^{(2)} = \frac{1}{O_2I\sin\psi_c} \qquad \kappa_{II}^{(2)} = 0 \tag{2.3.12}$$

where $O_2I = \frac{N_2}{2p}$ is the radius of the gear pitch circle. The positive sign of the curvature indicates that the curvature center lies on the positive direction of the surface unit normal. The unit vector $\underset{\sim}{k}_2$ in Eq. (2.3.5) coincides with the unit vector of $\underset{\sim}{\omega}^{(2)}$. Thus $\underset{\sim}{k}_2 = \underset{\sim}{k}_f$ where $\underset{\sim}{k}_f$ is the unit vector of the z_f-axis (Fig. 2.2.1).

Equations (2.3.1) - (2.3.12) yield the following relation between the derivative $m_{21}' = \frac{d}{d\phi_1}(m_{21}(\phi_1))$ and the principal curvatures of the crowned pinion tooth surface

$$m_{21}' = -\frac{A}{B}$$

where

$$A = |\kappa_I^{(1)}| - \frac{2p}{N_1\sin\psi_c}$$

$$B = \frac{N_2}{N_1\tan\psi_c}\left[\frac{2p}{N_1\sin\psi_c} + \frac{N_2\left(|\kappa_I^{(1)}| - \frac{2p}{N_1\sin\psi_c}\right)}{N_1+N_2}\right] \tag{2.3.13}$$

The goal is to provide a parabolic type of the transmission error function at least in the neighborhood of the main contact point I.

This requirement can be satisfied with the negative value of m'_{21}. Using Eq. (2.3.13), we can control the value of m'_{21} (it is negative) by varying the values of $\kappa_I^{(1)}$. It is evident that m'_{21} is negative if $\left|\kappa_I^{(1)}\right| > \frac{2p}{N_1 \sin\psi_c}$. Here: $\frac{2p}{N_1 \sin\psi_c}$ is the curvature at point I if the pinion would be provided with a regular involute curve; $\kappa_I^{(1)}$ is the curvature of the modified shape of the pinion tooth in its middle cross-section. We have emphasized that the pinion tooth surface must be deviated in the longitudinal direction (Fig. 1.3) to provide the point contact of the mating tooth surfaces and localize the bearing contact.

3. Design of the Crowned Pinion Tooth Surface for the Prescribed Transmission Error Function

3.1 Basic Concept

Consider that the function $\phi_2(\phi_1)$ that relates the angles of rotation of the gears is represented by (Fig. 3.3.1).

$$\phi_2(\phi_1) = \phi_1 \frac{N_1}{N_2} + b_1 - a_1\phi_1^2 \tag{3.1.1}$$

Here: $\phi_1 \frac{N_1}{N_2}$ is the linear part of function $\phi_2(\phi_1)$ and $\Delta\phi_2(\phi_1) = b_1 - a_1\phi_1^2$ is prescribed parabolic function of transmission errors.

The gear is provided with a regular involute surface. If the gears are aligned, the crowned pinion tooth surface and the gear tooth surface are in mesh in the middle cross-section. The contact of the gear tooth surface can be considered as the contact of two planar shapes. Our goal is to determine the shape of the pinion tooth surface in the middle cross-section.

We consider that the transmission errors in the region of $-\frac{\Pi}{N_1} < \phi_1 < \frac{\Pi}{N_1}$ are on average equal to zero. Thus

$$\int_{-\frac{\Pi}{N_1}}^{\frac{\Pi}{N_1}} (b_1 - a_1\phi_1^2)\, d\phi_1 = 0 \tag{3.1.2}$$

Equation (3.1.2) yields that

$$a_1 = \frac{e_1}{\left(\frac{\Pi}{N_1}\right)^2} \qquad b_1 = \frac{e_1}{3} \tag{3.1.3}$$

Henceforth we will use the following coordinate systems (Fig. 3.1.2): $S_1(X_1, Y_1)$ and $S_2(X_2, Y_2)$ that are rigidly connected to the pinion and gear respectively; and $S_f(x_f, y_f)$ that is the fixed coordinate system employed in the housing.

The determination of the pinion tooth shape is based on the following considerations:

(i) The gear tooth shape Σ_2 is a regular involute curve represented in coordinate system S_2 by the vector-function $\underset{\sim}{r}_2(\phi_G)$; the shape unit normal is represented by $\underset{\sim}{n}_2(\phi_G)$.

(ii) Using the coordinate transformation in transition from S_2 to S_f, we can represent the family of curves Σ_G in S_f as follows

$$[r_f^{(2)}] = [M_{f2}]\ [r_2] \qquad [n_f^{(2)}] = [L_{f2}]\ [n_2] \tag{3.1.4}$$

(iii) The equation of meshing is represented by

$$\underset{\sim}{n}_f^{(2)} \cdot \underset{\sim}{V}_f^{(21)} = 0 \tag{3.1.5}$$

where $\underset{\sim}{V}_f^{(21)}$ is the sliding velocity at the contact point.

Equations (3.1.4) and (3.1.5) yield the following relation

$$f(\phi_G, \phi_2) = 0 \tag{3.1.6}$$

that is called the equation of meshing.

(iv) The following equation

$$[r_1] = [M_{1f}][r_f^{(2)}] = [M_{1f}][M_{f2}][r_2] \tag{3.1.7}$$

and the equation of meshing (3.1.6) determine the shape of the pinion tooth in the middle cross-section. The surface of the pinion can be determined based on the shape in its middle cross-section.

3.2 Derivation of Pinion Tooth Surface

Gear Tooth Involute Shape Equations

The gear involute shape and its unit normal are represented in S_2 as follows:

$$x_2 = r_2[\sin\phi_G - \phi_G\cos\psi_c\cos(\phi_G-\psi_c)]$$

$$y_2 = r_2\ [\cos\phi_G + \phi_G\cos\psi_c\sin(\phi_G-\psi_c)] \qquad (3.2.1)$$

$$z_2 = 0$$

$$\underset{\sim}{n}_2 = \begin{bmatrix} -\cos(\phi_G-\psi_c) \\ \sin(\phi_G-\psi_c) \\ 0 \end{bmatrix} \qquad (3.2.2)$$

Here: r_2 is the radius of the gear pitch circle, ψ_c is the pressure angle and ϕ_G is the involute curve parameter; $\underset{\sim}{n}_2$ is the normal of tooth shape.

Matrix $[M_{f2}]$ is represented by

$$[M_{f2}] = \begin{bmatrix} -\cos\phi_2 & \sin\phi_2 & 0 & 0 \\ -\sin\phi_2 & -\cos\phi_2 & 0 & C^o \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.2.3)$$

Using Eqs. (3.1.4), (3.2.1), (3.2.2) and (3.2.3), we represent the family of shapes Σ_2 and the shape unit normal in S_f as follows

$$[r_f^{(2)}] = r_2 \begin{bmatrix} -\sin(\phi_G-\phi_2) + \phi_G\cos\psi_c\cos(\phi_G-\psi_c-\phi_2) \\ -\cos(\phi_G-\phi_2) - \phi_G\cos\psi_c\sin(\phi_G-\psi_c-\phi_2) \\ 0 \\ 1 \end{bmatrix} \tag{3.2.4}$$

$$[n_f^{(2)}] = \begin{bmatrix} \cos(\phi_G-\psi_c-\phi_2) \\ -\sin(\phi_G-\psi_c-\phi_2) \\ 0 \end{bmatrix} \tag{3.2.5}$$

Equation of Meshing

The sliding velocity $\underset{\sim}{V}^{(21)}$ is represented by the following equation

$$\underset{\sim}{V}^{(21)} = \underset{\approx}{\omega}^{(2)} \times (\overline{O_2O_1}+\underset{\sim}{r}_f^{(2)}) - \underset{\approx}{\omega}^{(1)}\times\underset{\sim}{r}_f^{(2)} \tag{3.2.6}$$

Here (Fig. 3.1.2)

$$\underset{\approx}{\omega}^{(1)} = \omega^{(1)}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \qquad \overline{O_2O_1} = \begin{bmatrix} 0 \\ -C^o \\ 0 \end{bmatrix} \qquad \underset{\approx}{\omega}^{(2)} = \omega^{(2)}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C^o = r_2 \left(\frac{N_1+N_2}{N_1}\right) \qquad \omega_2 = \omega_1\left(\frac{N_1}{N_2}\right) - 2a_1\phi_1$$

The expression for ω_2 is obtained by differentiating Eq. (3.1.1). Equation (3.2.6) yields

$$\underset{\sim}{V}^{(21)} = \begin{bmatrix} -Y_f^{(2)} \\ X_f^{(2)} \\ 0 \end{bmatrix} (\omega^{(2)}-\omega^{(1)}) + \begin{bmatrix} C^o \\ 0 \\ 0 \end{bmatrix} \omega^{(2)} \qquad (3.2.7)$$

Equations (3.1.5), (3.2.7), (3.2.4) and (3.2.5) yield the following equation of meshing

$$f(\phi_G,\phi_1) = \cos\psi_c\left(\frac{N_1}{N_2} + 1 - 2a_1\phi_1\right) - \left(\frac{N_1}{N_2} + 1\right)\cos(\phi_G-\psi_c-\phi_2) = 0 \qquad (3.2.8)$$

where $\phi_2 = \frac{N_1}{N_2}\phi_1 + (b_1-a_1\phi_1^2)$.

Fixing ϕ_1, we can obtain parameter ϕ_G. Equation (3.2.8) provides two solutions for ϕ_G. Therefore it is necessary to choose the solution corresponding to the working part of shape Σ_2.

Pinion Tooth Shape Equations

Matrix $[M_{1f}]$ is represented by (Fig. 3.1.2)

$$[M_{1f}] = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.2.9)$$

Equations (3.2.9), (3.1.7), (3.2.4) and (3.2.8) yield the following equations for the pinion shape

$$x_1 = -C^o \sin\phi_1 + r_2 \sin[\phi_1 - \psi_c + g(\phi_1)] \tag{3.2.10}$$

$$+ r_2 [\frac{N_1}{N_2} \phi_1 + b_1 - a_1 \phi_1^2 + \psi_c - g(\phi_1)] \cos\psi_c \cos[(\phi_1 + g(\phi_1))]$$

$$y_1 = C^o \cos\phi_1 - r_2 \cos[\phi_1 - \psi_c + g(\phi_1)]$$

$$+ r_2 [\frac{N_1}{N_2} \phi_1 + b_1 - a_1 \phi_1^2 + \psi_c - g(\phi_1)] \cos\psi_c \sin[\phi_1 + g(\phi_1)]$$

$$z_1 = 0$$

Here:

$$g(\phi_1) = \text{arc} \cos\{[1 - \frac{2a_1\phi_1}{(N_1/N_2)+1}] \cos\psi_c\}$$

Equations (3.2.10) represent the pinion shape in parametric form, with the parameter ϕ_1. For the case where the prescribed transmission errors are zero, $a_1 = 0$, $b_1 = 0$, $g(\phi_1) = \psi_c$ and Eq. (3.2.10) represent the pinion tooth shape as a regular involute shape.

Pinion Tooth Surface Equations

The crowned pinion tooth surface must be deviated from a cylindrical surface in the longitidunal direction (Fig. 1.3). The simplest way to obtain such deviation is based on representation of the pinion tooth surface as a surface of revolution. Such a surface can be generated by the rotation of the pinion tooth shape about a fixed axis. The determination of the pinion surface of revolution can be performed by using the following procedure:

(i) Assume that the pinion tooth shape has been transmitted from the coordinate system S_1 to the auxiliary coordinate system S_a (Fig. 3.2.1a). Thus the tooth shape is now rigidly connected to coordinate system S_a and is represented in S_a by the following matrix equation:

$$[r_a] = [M_{a1}][r_1(\phi_1)] \tag{3.2.11}$$

where

$$[M_{a1}] = \begin{bmatrix} 1 & 0 & 0 & R_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.12}$$

(ii) On the second stage we use a fixed coordinate S_b and rotate the coordinate system S_a with the tooth shape about the y_b-axis. The tooth shape will generate a surface of revolution represented in S_b as follows (Fig. 3.2.1b)

$$[r_b] = [M_{ba}][r_a] \tag{3.2.13}$$

where

$$[M_{ba}] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.14}$$

(iii) Now, we can represent the generated pinion tooth surface in coordinate system S_1 using the following matrix equation (Fig. 3.21c):

$$[r_1] = [M_{1b}][r_b] \tag{3.2.15}$$

where

$$[M_{1b}] = \begin{bmatrix} 1 & 0 & 0 & -R_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.2.16}$$

The final expression of the pinion tooth surface can be represented now as follows

$$[r_1(\theta,\phi_p)] = \begin{bmatrix} f_1(\theta,\phi_p) \\ f_2(\theta,\phi_p) \\ f_3(\theta,\phi_p) \\ 1 \end{bmatrix} = [M_{1b}][M_{ba}(\theta)][M_{a1}][r_1(\phi_P)] \tag{3.2.17}$$

Equations (3.2.11) - (3.2.17) yield

$$f_1(\theta,\phi_p) = x_1(\phi_p)\cos\theta - R(1-\cos\theta)$$

$$f_2(\theta,\phi_p) = y_1(\phi_p)$$

$$f_3(\theta,\phi_p) = x_1(\phi_p)\sin\theta + R\sin\theta \tag{3.2.18}$$

Here $x_1(\phi_p)$, $y_1(\phi_p)$ are functions (Eq. 3.2.10) that represent the pinion shape in the middle cross-section. Also for convenience, we use ϕ_p as curve parameter instead of ϕ_1.

3.3 Simulation of Meshing and Bearing Contact

The authors have developed the TCA (Tooth Contact Analysis) program to simulate the meshing and bearing contact of misaligned gears (see the Appendix). The results of the TCA program confirm that the proposed method of crowning reduces the sensitivity of gears to the misalignment (as shown in numerical example).

Example

Given: numbers of teeth: $N_1 = 20$, $N_2 = 40$; diametral pitch $P = 10 \frac{1}{in}$; pressure angle $\psi_c = 20^0$. The pinion tooth surface has been designed as a crowned surface and the function of transmission errors for the gears without misalignments has been represented as a parabolic function with the "Level of Kinematical Errors (LKE)" = 2 arc second. Radius of rotation $R_1 = 350$ in (see Fig. 3.2.1). The developed TCA program has been applied for the evaluation of transmission errors for the following misalignments:

(i) The change of the center distance is $\frac{\Delta c}{c} = 1\%$. The gear axes are not parallel but crossed and the twist angle is 5 arc minutes. The function of kinematic errors caused by the misalignments mentioned above is of a parabolic type and the maximum value of transmission errors is 1.2 arc seconds.

(ii) The gear axes are not parallel but intersected and form the angle α = 5 arc minutes. The function of kinematic errors is of a parabolic type and the maximum value of transmission errors is 2.0 arc seconds.

4. Deviation of Pinion Tooth Surface by Tool with Cone or Revolute Surface

4.1 Introduction

A new method for generatation of the pinion of spur gearing is developed. The involute spur gears with the crowned pinion tooth surface are less sensitive to the gear misalignment, the bearing contact has a favorable location and the kinematical errors are very small and with favorable shape. The crowning may be done by grinding or shaving using a tool with conic surface, a surface of revolution or a planar surface. The new method covers: (i) the theory of the proposed method of crowning, (ii) determination of the pinion surface generated by the tool, (iii) conditions of non-undercutting, (iv) principal curvatures and directions of the pinion tooth surface, (v) dimensions and orientation of the instantaneous contact ellipse, (vi) the bearing contact and (vii) the kinematical errors due to gear misalignment. Computer programs are developed in all stages including the simulation of meshing and bearing contact TCA (Tooth Contact Analysis) program.

4.2 Principle of Generation and Coordinate Systems

Consider two rigidly connect generating surfaces Σ_G and Σ_p. The generating surface is a plane and generates the gear tooth surface Σ_2 that is a regular involute surface. Surface Σ_p is a surface of revolution. Initially we consider that Σ_p is a cone surface and Σ_p and Σ_G contact each other along a straight line that is the generatrix of the cone.

Figure 4.2.0 shows the generating surfaces Σ_G and Σ_p. Figure 1.4

shows a sketch of the tool that could be used for pinion generation. Figure 4.2.1 illustrates the process of generation. While the rigidly connected generating surfaces perform a translational motion, the pinion and gear rotate about their axes 0_1 and 0_2 respectively. The parameter of motion of the cutters, s, and the angles of rotation of the pinion and the gear, ϕ_p and ϕ_G, are related as follows

$$s = r_1\phi_p = r_2\phi_G \tag{4.2.1}$$

where r_1 and r_2 are the gear centrodes radii; the cutter centrode is the straight line that is tangent to the gear centrodes. Point I is the instantaneous center of rotation. Coordinate systems S_1 and S_2 are rigidly connected to the pinion and the gear respectively. The rigidly connected tool surfaces (see Fig. 4.2.0) are represented in coordinate system S_c; S_f is the fixed coordinate system.

The generating surface Σ_G (a plane) is covered with the set of contact lines L_{G2} -- the instantaneous lines of tangency of surfaces Σ_G and Σ_2 (Fig. 4.2.2a). The location of these lines depends on the value of parameter ϕ_G. Line L_{GP} is the line of tangency of generating surfaces Σ_G and Σ_P. Points M_1, M_2,..., M_n represent the set of points of intersection of L_{GP} and L_{G2}. The coordinate transformation from S_c to S_2 is represented by the equation

$$[r_2] = [M_{2c}][r_c^{(G)}] \tag{4.2.2}$$

$$\underset{\sim}{n}^{(G)} \cdot \underset{\sim}{v}^{(G2)} = f_2(u_G, \theta_G, \phi_G) = 0 \tag{4.2.3}$$

Here: $\underset{\sim}{r}_c^{(G)}$ (U_G, θ_G) is the vector function that represents plane Σ_G ; $[M_{2c}]$ is the matrix that describes the coordinate transformation from S_c to S_2. Equation (4.2.3) is called the equation of meshing of the gear cutter and gear 2 [Litvin, 1987]. $\underset{\sim}{n}^{(G)}$ is the unit normal of generating surface and $\underset{\sim}{V}^{(C2)} = \underset{\sim}{V}^{(C)} - \underset{\sim}{V}^{(2)}$ is the relative velocity for the process of generation. Contact point M_i of surfaces Σ_1 and Σ_2 may be represented in coordinate system S_c as a function of parameter ϕ_G as follows

$$\underset{\sim}{r}_c^{(G)} = \underset{\sim}{r}_c^{(G)}(u_G, \theta_G) \qquad f_2(u_G, \theta_G, \phi_G) = 0 \tag{4.2.4}$$

$$F_2(U_G, \theta_G) = 0$$

Equations

$$\underset{\sim}{r}_c^{(G)} = \underset{\sim}{r}_c^{(G)}(u_G, \theta_G), \qquad F_2(u_G, \theta_G) = 0 \tag{4.2.5}$$

represent line L_{GP} in coordinate system S_c. Equations (4.2.4) represent the location of points $M_1, M_2, \ldots, M_n$ on surface Σ_G as a function of parameter ϕ_G. Matrix Eq. (4.2.2) and Eq. (4.2.4) represent the set of contact points of gear tooth surfaces Σ_2 and Σ_1 in coordinate system S_2. These points are the centers of the instantaneous contact ellipses that form the bearing contact of gear 2 and pinion 1.

Using the matrix equation

$$[r_f] = [M_{fc}][r_c^{(G)}] \tag{4.2.6}$$

and Eq. (4.2.4) we may represent in coordinate system S_f the line of action --- the set of contact points of Σ_1 and Σ_2 in the fixed coordinate system.

Similarly, considering the generating surface Σ_p, we may represent in coordinate system S_c the lines of contact, L_{P1}, of surfaces Σ_p and Σ_1 as follows

$$\underset{\sim}{r}_c^{(P)} = \underset{\sim}{r}_c^{(P)}(u_p, \theta_p) \qquad \underset{\sim}{n}^{(P)} \cdot \underset{\sim}{v}^{(P1)} = f_1(u_p, \theta_p, \phi_p) = 0 \tag{4.2.7}$$

Here $\underset{\sim}{n}^{(P)}$ is the unit normal of generating surface Σ_p; $\underset{\sim}{v}^{(P1)} = \underset{\sim}{v}^{(P)} - \underset{\sim}{v}^{(1)}$ is the relative velocity for the process of generation.

The pinion surface is represented by the equations

$$[r_1] = [M_{1c}][r_c^{(P)}] = [M_{1f}][M_{fc}][r_c^{(P)}] \qquad f_1(u_p, \theta_p, \phi_p) = 0 \tag{4.2.8}$$

Equations

$$\underset{\sim}{r}_c^{(P)} = \underset{\sim}{r}_c^{(P)}(u_p, \theta_p), \qquad F(u_p, \theta_p) = 0 \tag{4.2.9}$$

represent, in terms of u_p, and θ_p, the line of contact of surfaces Σ_p and Σ_G. in coordinate system S_c. Contact point M_i of surfaces Σ_1 and Σ_2 may be represented in coordinte system S_c as a function of parameter ϕ_p as follows

$$\underset{\sim}{r}_c^{(P)} = \underset{\sim}{r}_c^{(P)} (u_p, \theta_p) \qquad f_1(u_p, \theta_p, \phi_p) = 0 \tag{4.2.10}$$

$$F_1(u_p, \theta_p) = 0$$

Using the matrix equation

$$[r_f] = [M_{fc}][r_c^{(P)}] \tag{4.2.11}$$

and Eq. (4.2.10) we may represent the line of action of gears 1 and 2 as a function of ϕ_P in coordinate system S_f. Equations (4.2.4) and (4.2.6), and (4.2.10) and (4.2.11), determine the same line of action but in different parameter, ϕ_G or ϕ_P, respectively.

4.3 Tool Surface

The pinion generating surface is a cone and may be represented in an auxiliary coordinate system S_a as follows (Fig. 4.3.1)

$$\begin{aligned} x_a &= u_P \sin\alpha\cos\theta_P & y_a &= d - u_P\cos\alpha \\ z_a &= u_P \sin\alpha\sin\theta_P & 0 &< u_P < \frac{d}{\cos\alpha} \quad 0 < \theta_P < 2\Pi \end{aligned} \tag{4.3.1}$$

The surface normal is represented by

$$\underset{\sim}{N}_a = \frac{\partial \underset{\sim}{r}_a}{\partial \theta_P} \times \frac{\partial \underset{\sim}{r}_a}{\partial u_P} = u_P \sin\alpha \begin{bmatrix} \cos\alpha\cos\theta_P \\ \sin\alpha \\ \cos\alpha\sin\theta_P \end{bmatrix} \tag{4.3.2}$$

The unit surface normal is (provided $u_P\sin\alpha \neq 0$)

$$[n_a] = \begin{bmatrix} \cos\alpha\ \cos\theta_P \\ \sin\alpha \\ \cos\alpha\ \sin\theta_P \end{bmatrix} \tag{4.3.3}$$

Figure 4.3.2 illustrates the installment of the tool cone in coordinate system S_c. Axes of coordinate system S_b are parallel to axes of system S_a and axes x_b, y_b lie in plane $z_c = 0$. The coordinate transformation from S_a to S_c is represented by the following matrix equation

$$[r_c^{(P)}] = [M_{cb}][M_{ba}][r_a] = [M_{ca}][r_a] \tag{4.3.4}$$

Here

$$[M_{cb}] = \begin{bmatrix} \cos(\alpha-\psi_c) & \sin(\alpha-\psi_c) & 0 & 0 \\ -\sin(\alpha-\psi_c) & \cos(\alpha-\psi_c) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.3.5}$$

$$[M_{ba}] = \begin{bmatrix} 1 & 0 & 0 & -u_p^* \sin\alpha \\ 0 & 1 & 0 & -(d-u_p^* \cos\alpha) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.3.6}$$

Combining Eq. (4.3.5) and Eq. (4.3.6) results in:

$$[M_{ca}] = \begin{bmatrix} \cos(\alpha-\psi_c) & \sin(\alpha-\psi_c) & 0 & -u_p^* \sin\psi_c - d\sin(\alpha-\psi_c) \\ -\sin(\alpha-\psi_c) & \cos(\alpha-\psi_c) & 0 & u_p^* \cos\psi_c - d\cos(\alpha-\psi_c) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(4.3.7)

Here $u_P^* = |\overline{AO}_c| = \frac{d}{\cos\alpha} - \frac{b}{\cos\psi_c}$

Using Eqs. (4.3.4), (4.3.7) and (4.3.1), we represent the generating surface Σ_P in coordinate system S_c by equations as follows

$$[r_c^{(P)}] = \begin{bmatrix} u_P[\sin\alpha\cos\theta_P\cos(\alpha-\psi_c) - \cos\alpha\sin(\alpha-\psi_c)] - u_P^*\sin\psi_c \\ -u_P[\sin\alpha\cos\theta_P\sin(\alpha-\psi_c) + \cos\alpha\cos(\alpha-\psi_c)] + u_P^*\cos\psi_c \\ u_P\sin\alpha\sin\theta_P \\ 1 \end{bmatrix} \tag{4.3.8}$$

It is evident that the point of the cone surface with the surface parameter $u_P = u_P^*$ and $\theta_P = 0$ coincides with the origin 0_c of coordinate system S_c. The unit normal of cone surface is represented in coordinate system S_c by

$$[n_c^{(P)}] = [L_{ca}][n_a] \tag{4.3.9}$$

Here

$$[L_{ca}] = \begin{bmatrix} \cos(\alpha-\psi_c) & \sin(\alpha-\psi_c) & 0 \\ -\sin(\alpha-\psi_c) & \cos(\alpha-\psi_c) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.3.10}$$

Equations (4.3.9), (4.3.10) and (4.3.3) yield

$$[n_c^{(P)}] = \begin{vmatrix} \cos\alpha\cos\theta_P\cos(\alpha-\psi_c) + \sin\alpha\sin(\alpha-\psi_c) \\ -\cos\alpha\cos\theta_P\sin(\alpha-\psi_c) + \sin\alpha\cos(\alpha-\psi_c) \\ \cos\alpha\sin\theta_P \end{vmatrix} \quad (4.3.11)$$

4.4 Pinion Tooth Surface

The equation of meshing of the generating surface Σ_P and the pinion tooth surface is represented by

$$\underset{\sim}{n}^{(P)} \cdot \underset{\sim}{V}^{(P1)} = \underset{\sim}{n}^{(P)} \cdot (\underset{\sim}{V}^{(P)} - \underset{\sim}{V}^{(1)}) = 0 \quad (4.4.1)$$

We may also use the equation based on the condition that the contact normal must intersect the instantaneous axis of rotation I-I (Litvin, 1987). Thus we obtain (Fig. 4.2.1)

$$\frac{X_c - x_c^{(P)}}{n_{cx}^{(P)}} = \frac{Y_c - y_c^{(P)}}{n_{cy}^{(P)}} = \frac{Z_c - z_c^{(P)}}{n_{cz}^{(P)}} \quad (4.4.1a)$$

Here: (X_x, Y_c, Z_c) are the coordinates of a point that lies on axis I-I; $x_c^{(P)}$, $y_c^{(P)}$ and $z_c^{(P)}$ are the coordinates of the cone surface; n_{cx}, n_{cy} and n_{cz} are the projections of the surface unit normal. Here (Fig. 4.2.1)

$$X_c = s = r_1\phi_P \qquad Y_c = 0 \quad (4.4.2)$$

Equations (4.4.1a) (4.4.2), (4.3.8) and (4.3.11) yield

$$f_1(u_p,\theta_p,\phi_p) = r_1\phi_p[-\cos\alpha\cos\theta_p\sin(\alpha-\psi_c) + \sin\alpha\cos(\alpha-\psi_c)]$$

$$-u_p\cos\theta_p + u_p^*(\cos\theta_p\cos^2\alpha + \sin^2\alpha) = 0 \qquad (4.4.3)$$

Here (Fig. 4.3.2)

$$u_p^* = AO_c = \frac{d}{\cos\alpha} - \frac{b}{\cos\psi_c} \qquad (4.4.4)$$

The line of contact L_{PG} of generating surfaces Σ_P and Σ_G (Fig. 4.2.0) is generatrix of the cone surface determined with

$$\theta_p = 0 \qquad (4.4.5)$$

Equations (4.3.8), (4.4.3) and (4.4.5) represent the location of the instantaneous contact point of surfaces Σ_1 and Σ_2 on surface Σ_P as follows

$$x_c^{(P)} = r_1\phi_p\sin^2\psi_c \qquad y_c^{(P)} = -\ r_1\phi_p\sin\psi_c\cos\psi_c \qquad z_c^{(P)} = 0 \qquad (4.4.6)$$

The coordinate transformation in transition from S_c to S_f is represented by the matrix equation

$$[r_f] = [M_{fc}][r_c^{(P)}] \qquad (4.4.7)$$

where (Fig.4.2.1)

$$[M_{fc}] = \begin{bmatrix} 1 & 0 & 0 & -r_1\phi_P \\ 0 & 1 & 0 & r_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.4.8}$$

Equations (4.4.6) to (4.4.8) yield

$$x_f = r_1\phi_P\cos^2\psi_c \qquad y_f = r_1 - r_1\phi_P\sin\psi_c\cos\psi_c \qquad z_f = 0 \tag{4.4.9}$$

Equations (4.4.9) represent the line of action as a tangent to the base cylinder of radius r_{b1} that lies in plane $z_f = 0$ and passes through point I (Fig. 4.4.1).

The pinion tooth surface may be represented in coordinate system S_1 as the set of contact lines of surfaces Σ_P and Σ_1. Thus we obtain

$$[r_1] = [M_{1c}][r_c^{(P)}] = [M_{1f}][M_{fc}][r_c^{(P)}] \tag{4.4.10}$$
$$f_1(u_P, \theta_P, \phi_P) = 0$$

Here (Fig. 4.2.1)

$$[M_{1f}] = \begin{bmatrix} \cos\phi_P & \sin\phi_P & 0 & 0 \\ -\sin\phi_P & \cos\phi_P & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.4.11}$$

Equations (4.4.10), (4.4.3) and (4.3.8) yield

$$x_1 = u_P[\cos\theta_P \sin\alpha \cos(\alpha-\psi_c+\phi_P) - \cos\alpha \sin(\alpha-\psi_c+\phi_P)]$$
$$-u_P^* \sin(\psi_c-\phi_P) + r_1(-\phi_P\cos\phi_P+\sin\phi_P)$$

$$y_1 = -u_P[\cos\theta_P \sin\alpha \sin(\alpha-\psi_c+\phi_P) + \cos\alpha \cos(\alpha-\psi_c+\phi_P)]$$
$$+ u_P^* \cos(\psi_c-\phi_P) + r_1(\phi_P\sin\phi_P + \cos\phi_P)$$

$$z_1 = u_P \sin\phi_P \sin\alpha$$

$$f_1(u_P,\theta_P,\phi_P) = r_1\phi_P[-\cos\alpha\cos\theta_P\sin(\alpha-\psi_c) + \sin\alpha\cos(\alpha-\psi_c)]$$
$$-u_P\cos\theta_P + u_P^*(\cos\theta_P\cos^2\alpha + \sin^2\alpha) = 0 \qquad (4.4.12)$$

Equations (4.4.12) are equivalent to the representation of the pinion surface by the vector function $\underset{\sim}{r}_1(u_P,\theta_P,\phi_P)$ and the equation of meshing $f_1(u_P,\theta_P,\phi_P) = 0$. Taking it into account that $f_1(u_P,\theta_P,\phi_P) = 0$ is linear with respect to the parameter u_P, it is easy to eliminate u_P and represent the pinion surface in two parametric form, with the parameters, θ_P and ϕ_P.

The intersection of the surface by the plane $z_1 = 0$ represents a regular involute curve. Equations of this curve may be derived from the surface Eq. (4.4.12), taking it into account that

$$\theta_P = 0 \qquad \text{and} \qquad u_P-u_P^* = r_1\phi_P\sin\psi_c \qquad (4.4.13)$$

Thus, we have

$$x_1 = r_1[-\phi_P\cos\psi_c\cos(\phi_P-\psi_c) + \sin\phi_P] \qquad (4.4.14)$$
$$y_1 = r_1[\phi_P\cos\psi_c\sin(\phi_P-\psi_c) + \cos\phi_P]$$

Cutting the pinion tooth surface by a plane z_1 = const, we obtain the cross-section of the pinion tooth surface that is deviated from a regular involute curve (Fig. 4.4.2).

It will be shown later that the kinematic errors caused by the gear misalignment can be of two types that are shown in Fig. 1.2a and 1.2b. Kinematic errors shown in Fig. 1.2a correspond to the case when the transformation of rotation is interrupted and the gear tooth surfaces are out of contact. To avoid the appearance of such kinematic errors, a surface of revolution that slightly deviates from the cone surface is actually used. The determination of curvatures of the surface of revolution is a subject of optimization. Since a surface of revolution is used instead of a cone surface, the crowned pinion surface deviates from a regular involute surface in all cross-sections including $z_1 = 0$, the middle cross-section of the tooth surface.

4.5. Conditions of Pinion Without Undercutting

General Approach

The problem of undercutting of the pinion tooth surface by crowning is related with the appearance on the pinion tooth surface of singular points. It is known from differential geometry that the surface point is singular if the surface normal is equal to zero at such a point. Litvin, proposed a method (Litvin, 1987) to determine a line on the tool surface which will generate singular points on the surface that is generated by the tool. This line designated by L (Fig. 4.5.1) must be out of the working part of the tool surface to avoid undercutting of the pinion by crowning.

The limiting L of the tool surface is determined by the following equations

$$\underset{\sim}{r}_c^{(P)} = \underset{\sim}{r}_c^{(P)}(u_P, \theta_P) \tag{4.5.1}$$

$$f_1(u_P, \theta_P, \phi_P) = 0 \tag{4.5.2}$$

$$F(u_P, \theta_P, \phi_P) = 0 \tag{4.5.3}$$

Vector Eq. (4.5.1) represents the tool surface, Eq. (4.5.2) is the equation of meshing (see Eq. (4.4.3)) and Eq. (4.5.3) follows from the requirement that (Litvin, 1987)

$$\begin{vmatrix} \dfrac{\partial x_c^{(P)}}{\partial u_P} & \dfrac{\partial x_c^{(P)}}{\partial \theta_P} & -V_{cx}^{(C1)} \\ \dfrac{\partial y_c^{(P)}}{\partial U_P} & \dfrac{\partial y_c^{(P)}}{\partial \theta_P} & -V_{cy}^{(C1)} \\ \dfrac{\partial f_1}{\partial U_P} & \dfrac{\partial f_1}{\partial \theta_P} & -\dfrac{\partial f_1}{\partial \phi_P} \cdot \dfrac{d\phi_P}{dt} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial x_c^{(P)}}{\partial u_P} & \dfrac{\partial x_c^{(P)}}{\partial \theta_P} & -V_{cx}^{(C1)} \\ \dfrac{\partial z_c^{(P)}}{\partial U_P} & \dfrac{\partial z_c^{(P)}}{\partial \theta_P} & -V_{cz}^{(C1)} \\ \dfrac{\partial f_1}{\partial U_P} & \dfrac{\partial f_1}{\partial \theta_P} & -\dfrac{\partial f_1}{\partial \phi_P} \cdot \dfrac{d\phi_P}{dt} \end{vmatrix}$$

$$\begin{vmatrix} \dfrac{\partial y_c}{\partial U_P} & \dfrac{\partial y_c^{(P)}}{\partial \theta_P} & -V_{cy}^{(C1)} \\ \dfrac{\partial z_c^{(P)}}{\partial U_P} & \dfrac{\partial z_c^{(P)}}{\partial \theta_P} & -V_{cz}^{(C1)} \\ \dfrac{\partial f_1}{\partial U_P} & \dfrac{\partial f_1}{\partial \theta_P} & -\dfrac{\partial f_1}{\partial \phi_P} \cdot \dfrac{d\phi_P}{dt} \end{vmatrix} = 0 \tag{4.5.4}$$

Derivation of Equation

Relative velocity $\underset{\sim}{V}_c^{(C1)}$ is to be represented in coordinate system S_c as:

$$\underset{\sim}{V}_c^{(c1)} = \underset{\sim}{V}_c^{(c)} - \underset{\sim}{V}_c^{(1)} \qquad (4.5.5)$$

where $\underset{\sim}{V}_c^{(c)}$ is the velocity of the cutter and $\underset{\sim}{V}_c^{(1)}$ is the velocity of the pinion at generating point. Using the sketch in Fig. 4.2.1, we get

$$\underset{\sim}{V}_c^{(c)} = \begin{bmatrix} \frac{ds}{dt} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -r_1\omega_P \\ 0 \\ 0 \end{bmatrix} \qquad (4.5.6)$$

$$\underset{\sim}{V}_c^{(1)} = \underset{\approx}{\omega}_P \, x \underset{\sim}{r}_c + \overline{0_c 0_1} x \underset{\approx}{\omega}_P \qquad (4.5.7)$$

Deriving Eq. (4.5.7) we substituted the sliding vector $\underset{\approx}{\omega}_P$ that passes through 0_1 by the equal vector that passes through 0_c and the vector-moment $\overline{0_c 0_1} x \underset{\approx}{\omega}_P$.

Here:

$$\underset{\approx}{\omega}_P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega \qquad \overline{0_c 0_1} = \begin{bmatrix} r_1\phi_P \\ -r_1 \\ 0 \end{bmatrix} \qquad (4.5.8)$$

Equations from (4.5.6) to (4.5.8) yield

$$\underset{\sim c}{V}^{(C1)} = \begin{bmatrix} y_c^{(P)} \\ -x_c^{(P)}+r_1\phi_P \\ 0 \end{bmatrix} \omega \tag{4.5.9}$$

where $x_c^{(P)}$ and $y_c^{(P)}$ are represented by equations (4.3.8). Equations (4.5.9), (4.3.3), (4.4.3) and (4.5.4) yield

$$F_1(u_P, \theta_P) = m_1m_2q - (m_2+m_1m_4)qp - m_3m_5q+m_4p^2 = 0 \tag{4.5.10}$$

Here

$$q = \frac{u_P\cos\alpha}{d}$$

$$p = \frac{u_P^*\cos\psi_c}{d/\cos\alpha}$$

$$m_1 = \sin\alpha\sin(\alpha-\psi_c)\cos\theta_P + \cos\alpha\cos(\alpha-\psi_c)$$

$$m_2 = \sin\alpha\cos(\alpha-\psi_c) - \cos\alpha\sin(\alpha-\psi_c)\cos^3\theta_P$$

$$m_3 = [\sin\alpha\cos(\alpha-\psi_c) - \cos\alpha\sin(\alpha-\psi_c)\cos\theta_P]^3$$

$$m_4 = \sin\alpha\cos\alpha\sin^2\theta_P$$

$$m_5 = \frac{r_1\cos\alpha}{d}$$

Using Eq. (4.5.10) and taking it into account that $\frac{\partial F_1}{\partial \theta_P} \neq 0$, we can represent u_p as function (θ_P). Undercutting will be avoided if

$$u_p(\theta_p) > \frac{d}{\cos\alpha} \tag{4.5.11}$$

The analysis of Eq. (4.5.10) yields that inequality (4.5.11) is satisfied if the conditions of non-undercutting of the pinion generated by a regular rack cutter are satisfied.

4.6. Principal Directions and Curvatures of Tooth Surfaces

Introduction to Pinion Principal Directions and Curvatures

The direct determination of pinion principal directions and curvatures requires complicated derivations. A simplified approach for the solution to this problem has been proposed by Litvin, in 1969. The main idea is as follows:

Consider that the tool surface Σ_p generates surface Σ_1 as the envelope of the family of surfaces Σ_p. Let L_{p1} be the instantaneous line of contact of Σ_p and Σ_1 and M is a point of this line. We consider as given the principal curvatures and directions of Σ_p at any point and also the parameters of motion of surfaces Σ_p and Σ_1 being in mesh. Then the principal curvatures and directions of the generated surface Σ_1 can be expressed in terms of principal curvatures and directions of the generating surface Σ_p by using the equations that have been derived by Litvin, 1969.

Principal Curvatures and Directions of Σ_p

The tool surface is a cone surface and its principal directions coincide with the direction of the cone generatrix and the direction that is perpendicular to the cone generatrix. The Rodrigues' formula (Eq. 4.6.1) can be used to find the principal curvatures and directions.

$$\kappa_{I,II} \cdot \underset{\sim}{V}_r = -\dot{\underset{\sim}{n}}_r \tag{4.6.1}$$

Here: $\kappa_{I,II}$ are the principal curvatures of surface. $\underset{\sim}{V}_r$ is the velocity of a point that moves over a surface and $\dot{\underset{\sim}{n}}_r$ is the derivative of the surface unit normal $\underset{\sim}{n}$, when $\underset{\sim}{n}$ changes its direction due to the motion over the surface.

Equation (4.6.1) yields the following expressions for the principal curvatures and directions (see Eq. (4.3.1) for the tool cone surface Σ_P represented in coordinate system S_a)

$$\underset{\sim}{e}_I^{(P)} = \frac{\partial \underset{\sim}{r}_a}{\partial \theta_P} \div \left| \frac{\partial \underset{\sim}{r}_a}{\partial \theta_P} \right| = \begin{bmatrix} -\sin\theta_P \\ 0 \\ \cos\theta_P \end{bmatrix} \tag{4.6.2}$$

$$\kappa_I^{(P)} = -\frac{1}{u_P \tan\alpha} \tag{4.6.3}$$

$$\underset{\sim}{e}_{II}^{(P)} = \frac{\partial \underset{\sim}{r}_P}{\partial u_P} \div \left| \frac{\partial \underset{\sim}{r}_P}{\partial u_P} \right| = \begin{bmatrix} \cos\theta_P \sin\alpha \\ -\cos\alpha \\ \sin\theta_P \sin\alpha \end{bmatrix} \tag{4.6.4}$$

$$\kappa_{II}^{(P)} = 0 \tag{4.6.5}$$

The negative sign for κ_I indicates that the curvature center is located on the negative direction of the surface normal.

The principal curvatures are invariants with respect to the used coordinate system. The unit vectors of the principal directions, $\underset{\sim}{e}_I^{(P)}$ amd $\underset{\sim}{e}_{II}^{(P)}$, may be represented in coordinate system S_f by using the following matrix equation

$$[e_{I,II}^{(P)}]_f = [L_{fc}][L_{ca}][e_{I,II}^{(P)}]_a \qquad (4.6.6)$$

Here: $[e_{I,II}^{(P)}]_a$ and $[e_{I,II}^{(P)}]_f$ are vectors represented in S_a and S_f, respectively; matrix $[L_{ca}]$ is represented by Eq. (4.3.10); matrix $[L_{fc}]$ is the 3x3 unitary matrix (see matrix Eq. (4.4.8)).

Equation (4.6.6) yields

$$[e_I^{(P)}]_f = \begin{bmatrix} -\sin\theta_p\cos(\alpha-\psi_c) \\ \sin\theta_p\sin(\alpha-\psi_c) \\ \cos\theta_p \end{bmatrix} \qquad (4.6.7)$$

$$[e_{II}^{(P)}]_f = \begin{bmatrix} \cos\theta_p\sin\alpha\cos(\alpha-\psi_c) - \cos\alpha\sin(\alpha-\psi_c) \\ -\cos\theta_p\sin\alpha\sin(\alpha-\psi_c) - \cos\alpha\cos(\alpha-\psi_c) \\ \sin\theta_p\sin\alpha \end{bmatrix} \qquad (4.6.8)$$

Principal Curvatures and Directions of Pinion Tooth Surface

The determination of principal curvatures and directions for the pinion tooth surface is based on the following equations (see Litvin, 1987)

$$\tan 2\sigma^{(P1)} = -\frac{2b_{13}b_{23}}{b_{23}^2-b_{13}^2-(\kappa_I^{(P)}-\kappa_{II}^{(P)})b_{33}} \qquad (4.6.9)$$

$$\kappa_{II}^{(1)} - \kappa_{I}^{(1)} = \frac{b_{23}^2 - b_{13}^2 - (\kappa_I^{(P)} - \kappa_{II}^{(P)})b_{33}}{b_{33}\cos 2\sigma^{(P1)}} \tag{4.6.10}$$

$$\kappa_{II}^{(1)} + \kappa_{I}^{(1)} = \kappa_I^{(P)} + \kappa_{II}^{(P)} + \frac{b_{13}^2 + b_{23}^2}{b_{33}} \tag{4.6.11}$$

Here: $\kappa_I^{(P)}$ and $\kappa_{II}^{(P)}$ are the principal curvatures of the tool surface Σ_P; $\underset{\sim}{e}_I^{(P)}$ and $\underset{\sim}{e}_{II}^{(P)}$ are the unit vectors of the principal directions on Σ_P (Fig. 4.6.1); $\kappa_I^{(1)}$ and $\kappa_{II}^{(1)}$, $\underset{\sim}{e}_I^{(1)}$ and $\underset{\sim}{e}_{II}^{(1)}$ are the principal curvatures and directions on the pinion tooth surface Σ_1; and $\sigma^{(P1)}$ is measured counterclockwise from $\underset{\sim}{e}_I^{(P)}$ to $\underset{\sim}{e}_I^{(1)}$ (Fig. 4.6.1). The coefficients b_{13}, b_{23} and b_{33} have been derived from equations represented in [Litvin 1987] but modified for the case when a rack cutter generates a gear. The expressions for b_{13}, b_{23} and b_{33} are as follows

$$\begin{vmatrix} b_{13} \\ \\ b_{23} \end{vmatrix} = \begin{vmatrix} \underset{\sim}{e}_{II}^{(P)} \cdot \underset{\sim}{\omega}_P \\ \\ -\underset{\sim}{e}_I^{(P)} \cdot \underset{\sim}{\omega}_p \end{vmatrix} - \begin{vmatrix} -\kappa_I^{(P)} & 0 \\ \\ 0 & -\kappa_{II}^{(P)} \end{vmatrix} \begin{vmatrix} v_I^{(P1)} \\ \\ v_{II}^{(P1)} \end{vmatrix} \tag{4.6.12}$$

$$\begin{aligned} b_{33} = {} & [\underset{\sim}{n}\underset{\sim}{\omega}_p \underset{\sim}{V}_{tr}^{(P)}] + [\underset{\sim}{n}\underset{\sim}{\omega}_p V^{(P1)}] \\ & -\kappa_I^{(P)}(V_I^{(P1)})^2 - \kappa_{II}^{(P)}(V_{II}^{(P1)})^2 - \frac{\omega^2}{m_{1P}} m'_{1P}\, [\underset{\sim}{n}\underset{\sim}{r}^{(P)}\underset{\sim}{k}_f] \end{aligned} \tag{4.6.13}$$

where $m_{1P} = \dfrac{V_1}{\dot{s}} = \dfrac{r_1\omega_P}{\dot{s}}$

The vectors that are used in Eqs. (4.6.12) and (4.6.13) are represented in coordinate system S_f (Fig. 4.2.1); $\underset{\sim}{i}_f$, $\underset{\sim}{j}_f$ and $\underset{\sim}{k}_f$ are the unit vectors of coordinate axes of system S_f. The expressions of these vectors are as follows

$$\underset{\sim}{\omega}_p = \omega \underset{\sim}{k}_f \tag{4.6.14}$$

is the angular velocity of the pinion being in mesh with the rack cutter

$$V_{tr}^{(P)} = -\omega r_1 \underset{\sim}{i}_f \tag{4.6.15}$$

is the transfer velocity of a point of the rack cutter that performs translational motion; r_1 is the radius of the pinion centrode

$$\underset{\sim}{V}_{tr}^{(1)} = \underset{\sim}{\omega}_p \times \underset{\sim}{r}^{(P)} = \begin{vmatrix} i & j & k \\ 0 & 0 & \omega \\ x_f & y_f & z_f \end{vmatrix} = \omega \begin{bmatrix} -y_f \\ x_f \\ 0 \end{bmatrix} \tag{4.6.16}$$

is the transfer velocity of the pinion. The term, "transfer velocity" means that the velocity of a point that is rigidly connected to the tooth surface is considered. Such a point performs the motion with the tooth surface

$$\underset{\sim}{V}^{(P1)} = \underset{\sim}{V}^{(P)} - \underset{\sim}{V}^{(1)} = \omega \begin{bmatrix} y_f - r_1 \\ -x_f \\ 0 \end{bmatrix} \tag{4.6.17}$$

is the "sliding" velocity--the velocity of a point of the rack cutter with respect to the same point of the pinion. Vector $\underset{\sim}{V}^{(P1)}$ can be represented in terms of components x_c and y_c using matrix Eq. (4.4.8) that describes the coordinate transformation from S_c to S_f. Then we obtain

$$x_f = x_c - r_1\phi_P \qquad y_f = y_c + r_1 \tag{4.6.18}$$

Equations (4.6.17) and (4.6.18) yield

$$\underset{\sim}{V}^{(P1)} = \omega \begin{bmatrix} y_c \\ -x_c+r_1\phi_P \\ 0 \end{bmatrix} \tag{4.6.19}$$

The principal curvatures of the cone that is rigidly connected to the rack cutter are represented as

$$\kappa_I^{(P)} = -\frac{1}{u_P\tan\alpha} \qquad \kappa_{II}^{(P)} = 0 \tag{4.6.20}$$

The unit vectors of principal directions are designated by $\underset{\sim}{e}_I^{(P)}$ and $e_{II}^{(P)}$. Components $V_I^{(P1)}$ and $V_{II}^{(P1)}$ are represented as follows

$$V_I^{(P1)} = \underset{\sim}{V}^{(P1)}\cdot\underset{\sim}{e}_I^{(P)} \qquad V_{II}^{(P1)} = \underset{\sim}{V}^{(P1)}\cdot\underset{\sim}{e}_{II}^{(P)} \tag{4.6.21}$$

After substitution we obtain

$$b_{13} = \frac{\omega\sin\theta_P}{\sin\alpha}\left[1 - \frac{u_P^*}{u_P}\cos^2\alpha + \frac{r_1\phi_P}{u_P}\cos\alpha\sin(\alpha-\psi_c)\right] \tag{4.6.22}$$

$$b_{23} = -\omega\cos\theta_P \tag{4.6.23}$$

$$b_{33} = \omega^2(k_1+k_2+k_3+k_4) \tag{4.6.24}$$

where

$$k_1 = u_P\sin^2\theta_P\cos\alpha\sin\alpha - u_P^*\sin\alpha\cos\alpha(1-\cos\theta_P) \tag{4.6.25}$$

$$k_2 = r_1\phi_P[\cos\theta_P\cos\alpha\cos(\alpha-\psi_c) + \sin\alpha\sin(\alpha-\psi_c)] \tag{4.6.26}$$

$$k_3 = r_1[\cos\theta_P\cos\alpha\sin(\alpha-\psi_c) - \sin\alpha\cos(\alpha-\psi_c)] \tag{4.6.27}$$

$$k_4 = \frac{\sin^2\theta_P}{u_P\tan\alpha}[(u_P^*-u_P)\cos\alpha - r_1\phi_P\sin(\alpha-\psi_c)]^2 \tag{4.6.28}$$

For the contact point located in the middle section of the pinion we have $\theta_P = 0$ and

$$b_{13} = 0 \qquad b_{23} = \omega \qquad b_{33} = \omega^2[r_1\phi_P\cos\psi_c - r_1\sin\psi_c] \tag{4.6.29}$$

Then we obtain the following expressions for the pinion principal curvatures and directions

$$\sigma^{(P1)} = 0 \qquad \kappa_I^{(1)} = \kappa_I^{(P)} = -\frac{1}{u\tan\psi_c}$$

$$\kappa_{II}^{(1)} = \kappa_{II}^{(P)} - \frac{1}{r_1 \sin\psi_c - r_1 \phi_P \cos\psi_c} = \frac{-1}{r_1 \sin\psi_c - r_1 \phi_P \cos\psi_c}$$

$$\underset{\sim}{n} = \underset{\sim}{n}^{(P)} \qquad \underset{\sim}{e}_I^{(1)} = \underset{\sim}{e}_I^{(P)} \qquad \underset{\sim}{e}_{II}^{(1)} = \underset{\sim}{e}_{II}^{(P)} \tag{4.6.30}$$

The principle curvature $\kappa_{II}^{(1)}$ is the same as the principal curvature of a regular involute surface. However $\kappa_I^{(1)}$ differs from zero because the pinion is generated by a cone but not a plane as in the case of a regular involute surface.

Gear Principal Curvatures and Directions

The determination of the gear principal curvatures is based on the relation between the principal curvatures for spur involute gears. Figure 4.6.2 shows that two mating involute shapes are in tangency at point M. Let us designate the principal curvature for the pinion by $\kappa_I^{(1)}$ and $\kappa_{II}^{(1)}$ where (see Eq. 4.6.29)

$$\kappa_I^{(1)} = -\frac{1}{u \tan\psi_c} \qquad \kappa_{II}^{(1)} = -\frac{1}{r_1 \sin\psi_c - r_1 \phi_P \cos\psi_c} \tag{4.6.31}$$

The negative sign of $\kappa_I^{(1)}$ and $\kappa_{II}^{(1)}$ means that the radii of curvatures are opposite to the direction of the surface normal. The principal gear curvature $\kappa_I^{(2)} = 0$. The radii of curvature of spur involute gears are related as follows (Fig. 4.6.2) where $\kappa_{II}^{(1)} = -\frac{1}{r_1 \sin\psi_c - r_1 \phi_P \cos\psi_c}$

$$\frac{1}{\left|\kappa_{II}^{(1)}\right|} + \frac{1}{\kappa_{II}^{(2)}} = KL = \frac{r_{b1} + r_{b2}}{\tan\psi_c} = \frac{r_1 + r_2}{\sin\psi_c} = \frac{N_1 + N_2}{2P \sin\psi_c} \tag{4.6.32}$$

Then we obtain

$$\frac{1}{\kappa_{II}^{(2)}} = \frac{r_1+r_1}{\sin\psi_c} - \frac{1}{\left|\kappa_{II}^{(1)}\right|} = \frac{N_1+N_2}{2p\sin\psi_c} - \frac{1}{\left|\kappa_{II}^{(1)}\right|} \qquad (4.6.33)$$

The angle $\sigma^{(21)}$ between the principal curvatures is equal to zero.

4.7. Contact Ellipse and Bearing Contact

The dimensions and orientation of the instantaneous contact ellipse can be determined on the basis of equations that have been developed by Litvin (Litvin et al. 1982, and Litvin 1987). The input data for the computation is: $\kappa_I^{(i)}$, $\kappa_{II}^{(i)}$ $(i=1,2)$ $\sigma^{(12)}$ and ε where $\kappa_I^{(i)}$ and $\kappa_{II}^{(i)}$ are the principal curvatures, $\sigma^{(12)}$ is the angle formed by the unit vectors of principal directions, $\underset{\sim}{e}_I^{(1)}$ and $e_I^{(2)}$ and ε is the elastic deformation of the contacting surfaces at contact point. The bearing contact is formed by the set of instantaneous contact ellipses that move over the gear tooth surfaces in the process of meshing. In our case, when the crowned pinion is meshing with a gear without misalignment, $\underset{\sim}{e}_I^{(1)} = \underset{\sim}{e}_I^{(2)}$ and $\sigma^{(12)} = 0$ (Fig. 4.7.1). The axes of the contact ellipse are directed along the η-axis and ξ-axis , respectively. Generally the orientation of the contact ellipse is determined with the angle α_1. In the case where $\sigma^{(12)} = 0$, α_1 is equal to zero. The equations to be used for the computation are as follows

$$A = \frac{1}{4}\,[\kappa_\varepsilon^{(1)}-\kappa_\varepsilon^{(2)} - |g_1-g_2|] \qquad (4.7.1)$$

$$B = \frac{1}{4} [\kappa_{\varepsilon}^{(1)} - \kappa_{\varepsilon}^{(2)} + |g_1 - g_2|] \qquad (4.7.2)$$

where

$$\kappa_{\varepsilon}^{(1)} = \kappa_{I}^{(1)} + \kappa_{II}^{(1)} \qquad \kappa_{\varepsilon}^{(2)} = \kappa_{I}^{(2)} + \kappa_{II}^{(2)} \qquad (4.7.3)$$

$$g_1 = \kappa_{I}^{(1)} - \kappa_{II}^{(1)} \qquad g_2 = \kappa_{I}^{(2)} - \kappa_{II}^{(2)} \qquad (4.7.4)$$

$$a^* = \left|\frac{\varepsilon}{A}\right|^{1/2} \qquad b^* = \left|\frac{\varepsilon}{B}\right|^{1/2} \qquad (4.7.5)$$

The equations for $\kappa_{I}^{(i)}$ and $\kappa_{II}^{(i)}$ (i=1,2,) have been represented in Section 4.6. Figure 4.7.2 illustrates the bearing contact for spur gears with the crowned pinion tooth surface.

4.8. Simulation of Meshing and Determination of Kinematical Errors

Coordinate Systems

The simulation of meshing is a part of the computer aided TCA program developed by the authors. The simulation of meshing is based on equations that provide the continuous tangency of contacting surfaces.

Henceforth we will use the following coordinate systems: (i) S_f that is rigidly connected to the frame and (ii) an auxiliary coordinate system S_h that is also rigidly connected to the frame, and (iii) systems S_1 and S_2 that are rigidly connected to the pinion and the gear, respectively. The origins and coordinates axes z_f and z_1 of coordinate systems S_f and S_1 coincide with each other (Fig. 4.8.1a).

Axis z_f is the axis of rotation of the pinion. Also, the origins and coordinate axes z_h and z_2 coincide with each other and z_h is the axis of gear rotation (Fig. 4.8.1b). The errors of assembly of the gears are simulated with the orientation and location of coordinate system S_h with respect to S_f. Figure 4.8.2 shows the orientation

(a) The gear axes are crossed (Fig. 4.8.2a)

$$[M_{fh}] = \begin{bmatrix} -\cos\Delta\gamma & 0 & \sin\Delta\gamma & 0 \\ 0 & -1 & 0 & C \\ \sin\Delta\gamma & 0 & \cos\Delta\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.8.1}$$

(b) The gear axes are intersected (Fig. 4.8.2b)

$$[M_{fh}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -\cos\Delta\delta & -\sin\Delta\delta & C \\ 0 & -\sin\Delta\delta & \cos\Delta\delta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.8.2}$$

Pinion Tooth Surface Equations

The pinion tooth surface has been represented in coordinate system S_1 by Eq. (4.4.12).The coordinate transformation form S_1 to S_f is represented as follows

$$[r_f^{(1)}] = [M_{f1}][r_1] \tag{4.8.3}$$

Here

$$[M_{f1}] = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 & 0 \\ -\sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4.8.4)$$

where ϕ_1 is the angle of rotation of the pinion being in mesh with the gear.

Equation (4.4.12) represent the pinion tooth surface in a three-parametric form in terms of parameters u_P, θ_P and ϕ_P where ϕ_P is the angle of rotation of the pinion being in mesh with the coned cutter. These parameters are related by the equation of meshing and only two of them are independent. It is easy to eliminate parameter u_P and represent the pinion tooth surface in S_1 in the two-parametric form. Thus:

$$\underset{\sim}{r}_1 = \underset{\sim}{r}_1(\phi_P, \theta_P) \qquad (4.8.5)$$

where ϕ_P and θ_P serve as the surface parameters. The pinion tooth surface unit normal can be represented in coordinate system S_f as

$$[n_f^{(1)}] = [L_{f1}]\ [n_1] \qquad (4.8.6)$$

The 3x3 matrix $[L_{f1}]$ can be developed from $[M_{f1}]$ by eliminating the last row and the last column. The surface unit normal, $\underset{\sim}{n}_1$, is represented in S_1 by the following matrix equation

$$[n_1] = [L_{1c}][n_c^{(P)}] = [L_{1f}][L_{fc}][n_c^{(P)}] \qquad (4.8.7)$$

Here: $\underset{\sim}{n}_c^{(P)}$ is represented by Eqs. (4.3.11), $[L_{fc}]$ is the 3x3 unitary matrix and $[L_{1f}]$ is the 3 x 3 submatrix of matrix $[M_{1f}]$ given by Eq. (4.4.11).

The final expression for $\underset{\sim}{n}_f^{(1)}$ is as follows

$$[n_f^{(1)}] = [L_{f1}][L_{1f}][L_{fc}][n_c^{(P)}] \qquad (4.8.8)$$

Here: $[L_{fc}]$ is the 3x3 unitary matrix $[I]$, but $[L_{f1}]$ $[L_1 f] \neq [I]$ because the elements of these matrices are expressed in different terms - ϕ_P and ϕ_1

Gear Tooth Surface Equations

The gear tooth surface is a regular involute tooth surface and may be represented in coordinate system S_2 by equations that are similar to Eq. (4.4.14)

$$\begin{aligned} x_2 &= r_2 \; [-\phi_G \cos\psi_c \cos(\phi_G - \psi_c) + \sin\phi_G] \\ y_2 &= r_2 [\phi_G \cos\psi_c \sin(\phi_G - \psi_c) + \cos\phi_G] \\ z_2 &= \lambda \end{aligned} \qquad (4.8.9)$$

where ϕ_G is the angle of rotation of the gear being in mesh with the rack cutter. The gear tooth surface unit normal is represented by the equations

$$\underset{\sim}{n}_2 = \frac{\underset{\sim}{N}_2}{|\underset{\sim}{N}_2|} \qquad \underset{\sim}{N}_2 = \frac{\partial \underset{\sim}{r}_2}{\partial \phi_G} \times \frac{\partial \underset{\sim}{r}_2}{\partial \ell} \qquad (4.8.10)$$

Equations (4.8.9) and (4.8.10) yield

$$[n_2] = \begin{bmatrix} -\cos(\phi_G-\psi_c) \\ \sin(\phi_G-\psi_c) \\ 0 \end{bmatrix} \tag{4.8.11}$$

To represent the gear tooth surface and its unit normal in coordinate system S_h we use the following matrix equations (Fig. 4.8.2)

$$[r_h^{(2)}] = [M_{h2}]\ [r_2] \qquad [n_h^{(2)}] = [L_{h2}][n_2] \tag{4.8.12}$$

$$[M_{h2}] = \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 & 0 \\ \sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.8.13}$$

where ϕ_2 is the angle of rotation of the gear being in mesh with the pinion. Equations (4.8.9), (4.8.11), (4.8.12) and (4.8.13) yield

$$x_h^{(2)} = -\ r_2\{\phi_G\cos\psi_c\cos[(\phi_2-\phi_G) + \psi_c)] + \sin(\phi_2-\phi_G)\}$$

$$y_h^{(2)} = -\ r_2\{\phi_G\cos\psi_c\sin[(\phi_2-\phi_G) + \psi_c)] - \cos(\phi_2-\phi_G)\}$$

$$z_h^{(2)} = \lambda \tag{4.8.14}$$

$$[n_h^{(2)}] = \begin{bmatrix} -\cos[(\phi_2-\phi_G) + \psi_c] \\ -\sin[(\phi_2-\phi_G) + \psi_c] \\ 0 \end{bmatrix} \qquad ((4.8.15)$$

These equations with parameters $\phi_2 = \phi_c = 0$, $z_h^{(2)} = \lambda = 0$ determine point M with coordinates $(0, r_2, 0)$ and the surface unit normal at M with components $(-\cos\psi_c, -\sin\psi_c, 0)$ (Fig. 4.8.3).

To determine the gear tooth surface equations and the surface unit normal in coordinate system S_f we use the following matrix equations

$$[r_f^{(2)}] = [M_{fh}][r_h^{(2)}] \qquad [n_f^{(2)}] = [L_{fh}][n_h^{(2)}] \tag{4.8.16}$$

Matrix $[M_{fh}]$ (see Eqs. (4.8.1) and (4.8.2)) describes the coordinate transformation from S_h to S_f for the case where the gear is misaligned with respect to the pinion. Figure 4.8.4 illustrates the case where the gears are not misaligned, the shortest distance of gear axes $C = r_1 + r_2$ and the tooth surfaces of the pinion and the gear contact each other at point M. The misalignment of the gear will not cause the surface contact at the tooth edge since the pinion tooth surface is crowned and deviates from a regular involute surface.

Equation of Tangency

The contact of gear tooth surfaces is simulated in the developed TCA program by the following equations

$$\underset{\sim}{r}_f^{(1)}(\phi_p, \theta_p, \phi_1) = \underset{\sim}{r}_f^{(2)}(\phi_G, \ell, \phi_2) \tag{4.8.17}$$

$$\underset{\sim}{n}_f^{(1)}(\phi_p, \theta_p, \phi_1) = \underset{\sim}{n}_f^{(2)}(\phi_G, \phi_2) \tag{4.8.18}$$

At the contact point vector Eq. (4.8.17) provides the equality of position vectors and vector Eq. (4.8.18) provides the equality of surface unit normals. Vector Eq. (4.8.17) provides three independent scalar equations that relate $\phi_P, \theta_P, \phi_1, \phi_G, \lambda$ and ϕ_2. However, vector Eq. (4.8.18) provides only two independent scalar equations since $\underset{\sim}{n}_f^{(1)}$ and $\underset{\sim}{n}_f^{(2)}$ are unit vectors and $|\underset{\sim}{n}_f^{(1)}| = |\underset{\sim}{n}_f^{(2)}| = 1$. Thus Eqs. (4.8.17) and (4.8.18) yield five independent scalar equations as follows

$$f_i(\phi_P, \theta_P, \phi_1, \phi_G, \lambda, \phi_2) = 0 \qquad (i=1,2,\ldots,5) \tag{4.8.19}$$

Equation system (4.8.19) is the expression in implicit form of functions of one variable, for instance of ϕ_1. The Theorem of Implicit Function System Existence states:

(i) Consider that a set of parameters

$$P^o = (\phi_P, \theta_P, \phi_1, \phi_G, \lambda, \phi_2) \tag{4.8.20}$$

satisfies Eq. (4.8.19) and that the Jacobian

$$J = \frac{D(f_1, f_2, f_3, f_4, f_5)}{D(\phi_P, \theta_P, \phi_G, \lambda, \phi_2)} \neq 0 \tag{4.8.21}$$

(the partial derivatives are taken at point P^o).

(ii) Then, equation system (4.8.19) can be solved in the neighborhood of point P^o by functions

$$\phi_2(\phi_1), \qquad \phi_P(\phi_1), \qquad \theta_P(\phi_1), \qquad \phi_G(\phi_1), \qquad \lambda(\phi_1) \varepsilon C \tag{4.8.22}$$

The designation C^1 means that the functions have continuous derivatives at least of the first order. Knowing function $\phi_2(\phi_1)$, we can determine the function of kinematical errors

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \tag{4.8.23}$$

4.9 Modification of Generating Surface Σ_P

Basic Relations

The proposed method of crowning provides conjugated gear tooth surfaces and a localized bearing contact. Using the TCA program it can be proven that the kinematical errors caused by the gear misalignment are on a very low level. However, the shape of the function of kinematic errors is unfavorable, similar to the shape that has been represented in Fig. 1.2a. This disadvantage can be avoided by the modification of generating surface Σ_P. The authors proposed to use a surface of revolution instead of a cone surface. Figure 4.9.1 shows the axial section of generating surface Σ_P that is an arc of the circle of radius ρ. Controlling the magnitude of ρ, we can provide the favorable shape of the function of kinematical errors (Fig. 1.2b) and also control the level of kinematical errors. The solution to this problem is based on the relation between the principal curvatures and directions for the contacting surfaces and the derivative $m_{21}' = \frac{d}{d\phi_1}(m_{21}\phi_1))$. This relation (Litvin, 1987) is represented as follows

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 0 \tag{4.9.1}$$

Consider that the tooth surfaces of a crowned pinion and a gear are in contact at point M that coincides with the instantaneous center of rotation (Fig. 4.8.4). It is assumed that the gears are not misaligned. In this case the coefficients of determinant (4.9.1) are expressed as follows

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} -\kappa_I^{(1)} + \kappa_I^{(2)} & 0 \\ 0 & -\kappa_I^{(1)} + \kappa_I^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} = \begin{bmatrix} -\underset{\sim}{e}_{II}^{(1)} \cdot \underset{\sim}{\omega}^{(12)} \\ \underset{\sim}{e}_I^{(1)} \cdot \underset{\sim}{\omega}^{(12)} \end{bmatrix}$$

$$b_{33} = -\underset{\sim}{n}\ [(\underset{\sim}{\omega}^{(1)} x \underset{\sim}{V}_{+r}^{(2)}) - (\underset{\sim}{\omega}^{(2)} x \underset{\sim}{V}_{+r}^{(1)}] + [\omega^{(1)}]^2 m_{21}'(\underset{\sim}{n} x \underset{\sim}{k}_2) \cdot (\underset{\sim}{r}_f^{(1)} - \underset{\sim}{R}_f) \tag{4.9.2}$$

where

$$m_{21} = \frac{d\phi_2}{d\phi_1}, \qquad m_{21}' = \frac{dm_{21}}{d\phi_1} = \frac{d}{d\phi_1}\left(\frac{d\phi_2}{d\phi_1}\right)$$

Equations (4.3.11), (4.6.8), (4.6.9) with $\theta_P = 0$ yield (see Fig. 8.4)

$$\underset{\sim}{n} = \begin{bmatrix} \cos\psi_c \\ \sin\psi_c \\ 0 \end{bmatrix}$$

$$\underset{\sim}{e}_I^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underset{\sim}{e}_{II}^{(2)} = \begin{bmatrix} \cos\psi_c \\ -\sin\psi_c \\ 0 \end{bmatrix}$$

$$\underset{\sim}{\omega}^{(12)} = \underset{\sim}{\omega}^{(1)} - \underset{\sim}{\omega}^{(2)} = - \omega^{(1)}(1+\frac{N_1}{N_2})\ \underset{\sim}{k}_f$$

where N_1 and N_2 are the number of teeth of the pinion and gear

$$\underset{\sim}{R}_f = (r_1+r_2)\underset{\sim}{j}_f$$

where r_1 and r_2 are the radii of the pitch circles

$$\underset{\sim}{r}_f^{(1)} = r_1\underset{\sim}{j}_f$$

$$\underset{\sim}{V}_{tr}^{(1)} = \omega^{(1)} x \underset{\sim}{r}_f^{(1)} = \omega^{(1)} r_1 \underset{\sim}{i}_f$$

$$\underset{\sim}{V}_{tr}^{(2)} = \underset{\sim}{\omega}^{(2)} x (\underset{\sim}{r}_f^{(1)} - \underset{\sim}{R}_f) = \omega^{(1)} r_1 \underset{\sim}{i}_f \tag{4.9.3}$$

Substituting Eqs. (4.9.3) into Eq. (4.9.2), we obtain

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} = \begin{bmatrix} -\kappa_I^{(1)} + \kappa^{(2)} & 0 \\ 0 & -\kappa_{II}^{(1)} + \kappa_{II}^{(2)} \end{bmatrix}$$

$$b_{13} = 0$$

$$b_{23} = -\omega^{(1)}(1+\frac{N_1}{N_2}) \tag{4.9.4}$$

$$b_{33} = \omega^{(2)} r_1 [(1+\frac{N_1}{N_2}) \sin\psi_c + m_{21}' \frac{N_2}{N_1} \cos\psi_c]$$

Equations (4.9.1) and (4.9.4) yield the following relation between the derivative m_{21}' and the principal curvatures

$$m_{21}' = \frac{(1+\frac{N_1}{N_2})^2 - (1+\frac{N_1}{N_2}) \sin\psi_c r_1 [-\kappa_{II}^{(1)} + \kappa_{II}^{(2)}]}{r_1 \cos\psi_c \frac{N_2}{N_1} [-\kappa_{II}^{(1)} + \kappa_{II}^{(2)}]} \tag{4.9.5}$$

Here (see Eqs. (4.6.31) and (4.6.32))

$$\kappa_{II}^{(1)} = \kappa_{II}^{(P)} - \frac{1}{r_1 \sin\psi_c} \qquad \kappa_{II}^{(2)} = \frac{1}{r_2 \sin\psi_c} = \frac{N_1/N_2}{r_1 \sin\psi_c} \tag{4.9.6}$$

$$m_{21}' = (1+\frac{N_1}{N_2}) \frac{N_1}{N_2} \tan\psi_c \frac{\kappa_{II}^{(P)}}{[(1+\frac{N_1}{N_2}) \frac{1}{r_1 \sin\psi_c} - \kappa_{II}^{(P)}]} \tag{4.9.7}$$

Here (Fig. 4.9.1)

$$\kappa_{II}^{(P)} = -\frac{1}{\rho}$$

where ρ is the radius of the arc circle that generates the surface of revolution.

There are two important particular cases for the application of Eq. (4.9.7) (i) The generating surface Σ_P is a cone. Considering that $\rho \to \infty$ and the generating surface becomes a cone, we obtain that $m'_{21} = 0$ if the gears are not misaligned. In this case, when misalignment exists, kinematic errors will occur as shown in Fig. 1.2a which are very undersirable. (ii) The generating surface Σ_P is a revolute surface, ρ is positive, $\kappa_{II}^{(P)} < 0$ $(|\kappa_{II}^{(P)}| = \frac{1}{\rho})$ and $m'_{21} < 0$ (m'_{21} is taken at point M). Then the function of kinematic errors will be similar to that as shown in Fig. 1.2b even when the gears are not aligned. It will be shown below that by choosing an appropriate value for ρ it is possible to keep the kinematical errors of misaligned gears on a very low level.

Surface of Revolution Equations

The surface of revolution is generated by an arc of circle of radius ρ. The arc parameters provide a common normal for the cone surface and the surface of revolution at their point of tangency M (Fig. 4.9.2a). The circular arc is expressed in the auxiliary coordinate system S_e as follows

$$
\begin{aligned}
x_e &= \rho[\cos(\alpha+\beta)-\cos\alpha] + u_P^{*}\sin\alpha \\
&= u_P^{*}\sin\alpha - 2\rho\sin^{\beta}(\alpha+\frac{\beta}{2}) \\[2ex]
y_e &= \rho[\sin(\alpha+\beta) - \sin\alpha] + \frac{b}{\cos\psi_c}\cos\alpha \\
&= \frac{b}{\cos\psi_c}\cos\alpha + 2\rho\sin\frac{\beta}{2}\cos(\alpha+\frac{\beta}{2}) \\[2ex]
z_e &= 0
\end{aligned}
\qquad (4.9.9)
$$

The unit normal to the arc at point M is also the normal to the cone surface and surface revolution at this point and it can be expressed as follows

$$[n_e] = \begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \\ 0 \end{bmatrix} \tag{4.9.10}$$

The surface of revolution is generated by rotation of the circular arc about the y_e-axis and may be represented in coordinate system S_a as follows

$$[r_a] = [L_{ae}][r_e] \qquad [n_a] = [Lae][n_e] \tag{4.9.11}$$

Here (Fig. 4.9.2b)

$$[L_{ae}] = \begin{bmatrix} \cos\theta_P & 0 & \sin\theta_P \\ 0 & 1 & 0 \\ -\sin\theta_P & 0 & \cos\theta_P \end{bmatrix} \tag{4.9.12}$$

Then we obtain

$$\begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = \begin{bmatrix} [U_P^* \sin\alpha - 2\rho\sin\frac{\beta}{2}\sin(\alpha+\frac{\beta}{2})]\cos\theta_P \\ \frac{b}{\cos\psi_c}\cos\alpha + 2\rho\sin\frac{\beta}{2}\cos(\alpha+\frac{\beta}{2}) \\ [U_P^* \sin\alpha - 2\rho\sin\frac{\beta}{2}\sin(\alpha+\frac{\beta}{2})]\sin\theta_P \end{bmatrix} \tag{4.9.13}$$

$$[n_a] = \begin{bmatrix} \cos\theta_P\cos(\alpha+\beta) \\ \sin(\alpha+\beta) \\ \sin\theta_P\cos(\alpha+\beta) \end{bmatrix} \tag{4.9.14}$$

The installment of the generating surface in coordinate system S_c is the same as it was described in Section 4.3. The generating surface and its unit normal are represented in S_c as follows

$$[r_c^{(P)}] = [M_{ca}][r_a] = \tag{4.9.15}$$

$$\begin{bmatrix} 2\rho\sin\frac{\beta}{2}\ [\sin(\alpha-\psi_c)\cos(\alpha+\frac{\beta}{2}) - \cos\theta_P\cos(\alpha-\psi_c)\sin(\alpha+\frac{\beta}{2})] \\ \quad + U_P^*\sin\alpha\cos(\alpha-\psi_c)(\cos\theta_P-1) \\ 2\rho\sin\frac{\beta}{2}\ [\cos(\alpha-\psi_c)\cos(\alpha+\frac{\beta}{2}) + \cos\theta_P\sin(\alpha-\psi_c)\sin(\alpha+\frac{\beta}{2})] \\ \quad - U_P^*\sin\alpha\sin(\alpha-\psi_c)(\cos\theta_P-1) \\ [U_P^*\sin\alpha - 2\rho\sin\frac{\beta}{2}\sin(\alpha+\frac{\beta}{2})]\sin\theta_P \end{bmatrix}$$

$$[n_c^{(P)}] = [L_{ca}]\ [n_a] =$$

$$\begin{bmatrix} \cos\theta_P\cos(\alpha+\beta)\cos(\alpha-\psi_c) + \sin(\alpha+\beta)\sin(\alpha-\psi_c) \\ -\cos\theta_P\cos(\alpha+\beta)\sin(\alpha-\psi_c) + \sin(\alpha+\beta)\cos(\alpha-\psi_c) \\ \sin\theta_P\cos(\alpha+\beta) \end{bmatrix} \tag{4.9.16}$$

Pinion Surface

The pinion surface is represented in coordinate system S_1 as follows

$$[r_1] = [M_{1f}][M_{fc}][r_c] \qquad f(\beta_1, \theta_P, \phi_P) = 0 \tag{4.9.17}$$

Here $f(\beta, \theta_P, \phi_P) = 0$ is the equation of meshing (see Section 4.4) represented as follows

$$f(\beta, \theta_P \phi_P) = r_1 \sin\beta \cos\theta_P - u_P^* \sin\alpha \sin(\alpha+\beta)(\cos\theta_P - 1)$$

$$- r_1 \phi_P \sin(\alpha+\beta)\cos(\alpha-\psi_c) - \cos\theta_P \cos(\alpha+\beta)\sin(\alpha-\psi_c) = 0 \tag{4.9.18}$$

TCA Program

Following the procedure of derivations described in Section 4.8, we can now develop the TCA program for the case where the generating surface Σ_P is a surface of revolution. Using this program, we can determine the kinematical errors caused by the gear.

Example

$P_n = 10 \qquad N_1 = 20, \qquad N_2 = 40 \qquad C = 1.01(r_1+r_2) \qquad \Delta\gamma = 5'$

$\psi_c = 20^0 \qquad \alpha = 30^0 \qquad \rho = 500 \text{ in} \qquad d = 0.176 \text{ in}$

The results of computation of kinematical errors are shown in Fig. 4.9.3. It was found that the function of kinematical errors has the desired shape, and the maximal value is 0.16 arc sec.

Conclusion

The authors have developed: (i) The basic principles for the optimal geometry of crowned spur gears; (ii) New methods for generation of crowned pinion tooth surface; (iii) Tooth contact analysis programs for the simulation of meshing and bearing contact.

The features of the optimal geometry of the crowned spur gears are as follows: (i) the gear is provided with a regular involute surface; (ii) the pinion tooth surface is deviated from a regular involute surface to provide a localized bearing contact and reduce the sensitivity of the gears to their misalignment; (iii) the function of transmission errors of the misaligned gears is of a parabolic type with the small level of maximal errors, less than 5 arc seconds.

The new methods for generation of the crowned pinion surface need simple tools that have to be provided with a generating plane or a surface of revolution that slightly deviates from a cone surface. The developed TCA programs provide the information on the bearing contact and the transmission errors for the misaligned gears.

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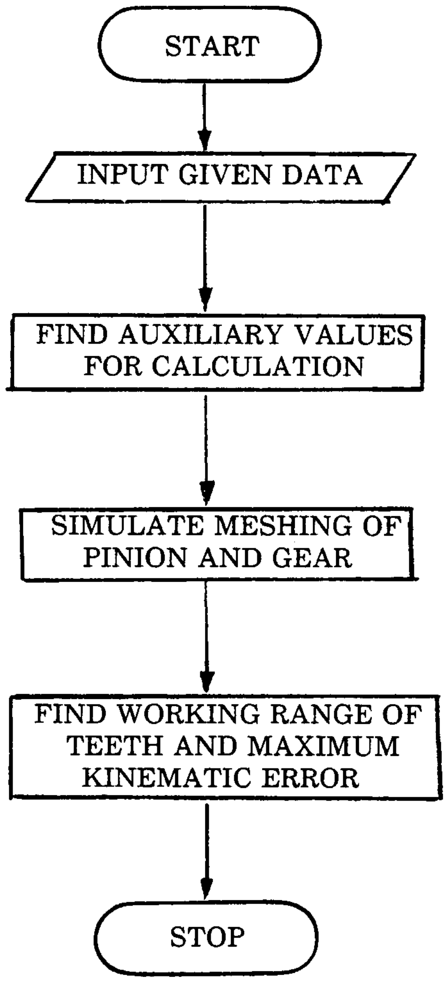
FLOWCHART FOR PROGRAM I



```
C... ********************************************************
C... *                                                      *
C... *                                                      *
C... *                     PROGRAM I                        *
C... * KINEMATIC ERROR OF A REGULAR INVOLUTE GEAR MESHING   *
C... *     WITH A PINION CROWNED BASED ON PREDESIGNED       *
C... *              KINEMATIC ERROR FUNCTION                *
C... *                                                      *
C... *               AUTHORS: FAYDOR LITVIN                 *
C... *                        JIAO ZHANG                    *
C... *                                                      *
C... ********************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE KINEMATIC ERROR OF A CROWNED
C     PINION MESHING WITH A REGULAR INVOLUTE GEAR. THE PINION IS
C     CROWNED BASED ON PREDESIGNED KINEMATIC ERROR FUNCTION.
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN77.  IT CAN BEE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Z(99),ERR(99),ERROR(99)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,C,
     +                S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R,CONST(9)
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C         EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C       NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NC=100
      DELTA=1.D-3
      EPSI=1.D-12
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      NDIM=10
      NE=5
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C          RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C          RKSI=PRESSURE ANGLE; HDC=HEIGHT OF DEDENDUM OF PINION;
C          COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)
```

```
C        R=RADIUS OF ROTATION AXIS FOR GENERATING PINION SURFACE
      PN=10.D0
      N1=20
      RMPG=2.D0
      RKSI=20.D0*DR
      HDC=1.D0/PN
      COE=1.010D0
      R=3.5D2
C    (2) PRE-DESIGNED FUNCTION: INPUT CONSTANTS HERE AND INPUT FUNCTION
C        IN SUBROUTINE FUNC
C        MATRIX CONST(J) IS USED FOR CONSTANTS OF FUNCTION
      CONST(1)=2.D0*3.929751681D-4
C    (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C        NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM 0. TO
C        (NG-1)*GAMMAI); GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
      NMIS=2
      NG=2
      GAMMAI=5.D0
C    (4) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREEE)
      FEEI=1.0D0*DR
C
C   DESCRIPTION OF OUTPUT PARAMETERS
C
C     FEE1=ROTATION ANGLE OF PINION
C     FEE2=ROTATION ANGLE OF GEAR
C     RP=RADIUS OF PINION CONTACT POINT
C     RG=RADIUS OF GEAR CONTACT POINT
C
C   FIND AUXILIARY VALUES FOR CALCULATION
      RP=FLOAT(N1)/2./PN
      RG=RP*RMPG
      C=(RP+RG)*COE
      CK=DCOS(RKSI)
      SK=DSIN(RKSI)
      NCOEF=360.D0*DR/FEEI/FLOAT(N1)+0.3D0
      N=NCOEF*2+1
      NT=N-NCOEF
      FII=360.D0/RMPG/FLOAT(N1)
C   DEFINE MATRIX DESCIBEING MISALIGNMENT AND OUTPUT ASSEMBLING CONDITION
      DO 5 I=1,3
      DO 5 J=1,3
      S(I,J)=0.D0
      IF (I.EQ.J) S(I,J)=1.D0
    5 CONTINUE
      DO 15 LL=1,NG
      GAMMA=GAMMAI*FLOAT(LL-1)/60.D0*DR
      CG=DCOS(GAMMA)
      SG=DSIN(GAMMA)
      GAMMA=GAMMA/DR*60.D0
      IF (NMIS.EQ.1) THEN
      WRITE (LP,10) COE,GAMMA
   10 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG);     CROSSING ANGLE=',
     +        F5.1,'(M)')
      S(1,1)=CG
      S(1,3)=-SG
      S(3,1)=SG
      S(3,3)=CG
      ELSE
```

```
      WRITE (LP,20) COE,GAMMA
   20 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG);     INTERSECTING ANGLE=',
     +        F5.1,'(M)')
      S(2,2)=CG
      S(2,3)=-SG
      S(3,2)=SG
      S(3,3)=CG
      END IF
C  SIMULATING MESHING OF PINION AND GEAR(L=1 FOR FINDING INITIAL
C    ROTATION CORRESPONDING TO 0 PINION ROTATION) AND OUTPUT RESULTS
      DO 25 L=1,2
      DO 35 I=1,4
   35 X(I)=0.D0
      DO 45 I=1,N
      X(7)=FEEI*FLOAT(I-(N+1)/2)
      IF (L.EQ.1) X(7)=0.D0
      X(5)=DARSIN(RP*X(7)*SK/R)
      X(2)=X(7)/RMPG
      SF=DSIN(X(7))
      CF=DCOS(X(7))
      CALL NONLIN
      X(8)=X(2)+X(3)
      IF (L.EQ.1) THEN
      XIN=X(8)
      WRITE (LP,30)
   30 FORMAT (////8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
     +        'RP',13X,'RG'/)
      GO TO 25
      END IF
      X(8)=X(8)-XIN
      X(7)=X(7)/DR
      X(8)=X(8)/DR
      X(9)=(X(8)-X(7)/RMPG)*3600.D0
      Z(I)=X(8)
      ERR(I)=X(9)
      WRITE (LP,40) (X(J),J=7,11)
   40 FORMAT (1X,5F15.7,F15.7)
   45 CONTINUE
      WRITE (LP,50)
   50 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:'/)
      DO 55 I=1,NT
      X(7)=FEEI*FLOAT(I-(N+1)/2)/DR/2.D0
      X(8)=X(7)+FII
      KK=I+NCOEF
      ANGLE=Z(KK)-Z(I)
      ERROR(I)=(ANGLE-FII)*3600.D0
      WRITE (LP,60) X(7),X(8),ANGLE
   60 FORMAT (1X,'(',F7.2,'----',F7.2,'):',F15.7,F15.7)
   55 CONTINUE
      DO 65 I=1,NT
      ATEMP2=ERROR(I)
      IF (I.NE.1) THEN
      IF (ATEMP1.GT.0.D0.AND.ATEMP2.LE.0.D0) GOTO 75
      END IF
      ATEMP1=ATEMP2
   65 CONTINUE
      WRITE (LP,70)
   70 FORMAT (//1X,'MESHING IS DISCONTINOUS')
```

```
      GOTO 25
   75 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
      EMAX=0.D0
      EMIN=0.D0
      DO 85 J=1,NCOEF
      KS=I+J-1
      ET=ERR(KS)
      IF (ET.LT.EMIN) EMIN=ET
      IF (ET.GT.EMAX) EMAX=ET
   85 CONTINUE
      ET=EMAX-EMIN
      KK=I+NCOEF
      WRITE (LP,80) Z(I),Z(KK),ET
   80 FORMAT (//1X,'WORKING RANGE FOR THE GEAR TOOTH: ',F7.2,'----',
     +     F7.2/1X,'THE MAXIMUM KINEMATIC ERROR: ',F15.7,'  (S)')
   25 CONTINUE
   15 CONTINUE
      WRITE (LP,90)
   90 FORMAT(1H1)
      STOP
      END
C
      SUBROUTINE FUNC
C
C  THIS SUBROUTINE IS USED TO GENERATE FIVE FUNCTIONS, THAT IS, AT
C    CONTACT POINT, POSITION VECTOR AND NORMAL OF PINION AND GEAR
C     MUST COINCIDE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,C,
     +                S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R,CONST(9)
C  INPUT PRE-DESIGN ERROR FUNCTION EF AND ITS DIFFEVERTIVE DEF,
C    RECALLING X(5) IS FEEP
      EF=-CONST(1)*X(5)*X(5)/2.D0
      DEF=-CONST(1)*X(5)
      CT=DCOS(X(4))
C
      ST=DSIN(X(4))
      CFEE=DCOS(X(3))
      SFEE=DSIN(X(3))
      CFEK=DCOS(X(3)+RKSI)
      SFEK=DSIN(X(3)+RKSI)
      GF=DARCOS((1.D0+DEF*RMPG/(1.D0+RMPG))*CK)
      CFGK=DCOS(X(5)+GF-RKSI)
      SFGK=DSIN(X(5)+GF-RKSI)
      CFP=DCOS(X(5))
      SFP=DSIN(X(5))
      CFPG=DCOS(X(5)+GF)
      SFPG=DSIN(X(5)+GF)
      T3=X(5)/RMPG+EF+RKSI-GF
      X11=-(RP+RG)*SFP+RG*SFGK+RG*T3*CK*CFPG
      Y11= (RP+RG)*CFP-RG*CFGK+RG*T3*CK*SFPG
      XP=X11*CT+R*(CT-1.D0)
      YP=Y11
      ZP=X11*ST+R*ST
      RNXP=CFPG*CT
      RNYP=SFPG
```

```
      RNZP=CFPG*ST
      XG=RG*SFEE+RG*X(2)*CK*CFEK
      YG=-RG*CFEE+RG*X(2)*CK*SFEK
      ZG=X(1)
      RNXG=CFEK
      RNYG=SFEK
      RNZG=0.D0
      Y(1)=CF*XP+SF*YP-S(1,1)*XG-S(1,2)*YG-S(1,3)*ZG
      Y(2)=CF*YP-SF*XP-S(2,1)*XG-S(2,2)*YG-S(2,3)*ZG-C
      Y(3)=ZP-S(3,1)*XG-S(3,2)*YG-S(3,3)*ZG
      Y(4)=CF*RNYP-SF*RNXP-S(2,1)*RNXG-S(2,2)*RNYG-S(2,3)*RNZG
      Y(5)=RNZP-S(3,1)*RNXG-S(3,2)*RNYG-S(3,3)*RNZG
      X(10)=DSQRT(XP*XP+YP*YP)
      X(11)=DSQRT(XG*XG+YG*YG)
C     WRITE (6,20) XP,YP,ZP,XG,YG,X(6),A1,A2,A3
C  20 FORMAT (1X,'$$$$',6F15.7)
      RETURN
      END
C
      SUBROUTINE NONLIN
C
C  THIS SUBROUTINE IS USED TO SOLVE NONLINEAR EQUATIONS BY NEWTON-
C    RAPHSON METHOD
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
C
      DO 5 I=1,NC
      CALL FUNC
C     WRITE (6,10) I,(X(J),Y(J),J=1,5)
C  10 FORMAT(1X,'***',I5/5(1X,2D15.7/))
      DO 15 J=1,NE
      IF (DABS(Y(J)).GT.EPSI) GO TO 25
   15 CONTINUE
      GO TO 105
   25 DO 35 J=1,NE
   35 Y1(J)=Y(J)
      DO 45 J=1,NE
      DIFF=DABS(X(J))*DELTA
      IF (X(J).EQ.0.D0) DIFF=DELTA
      XMAM=X(J)
      X(J)=X(J)-DIFF
      CALL FUNC
      X(J)=XMAM
      DO 55 K=1,NE
   55 A(K,J)=(Y1(K)-Y(K))/DIFF
   45 CONTINUE
      DO 65 J=1,NE
   65 Y(J)=-Y1(J)
      CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
      CALL SOLVE (NDIM.NE,A,Y,IPVT)
      DO 75 J=1,NE
   75 X(J)=X(J)+Y(J)
    5 CONTINUE
  105 RETURN
      END
C
C
```

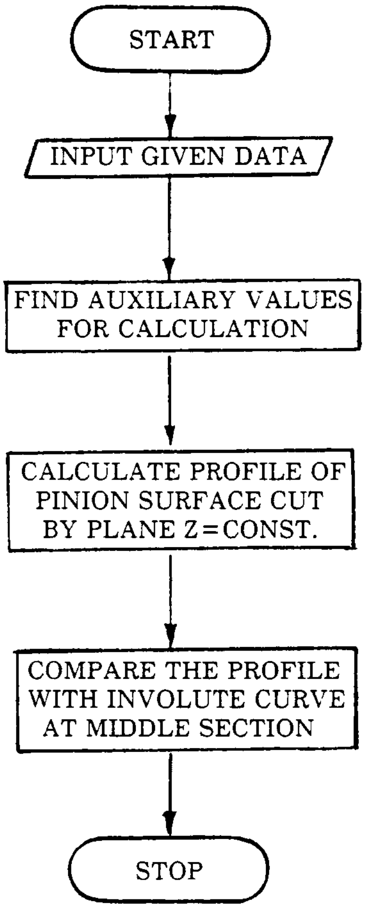
```
C
      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C  DECOMPOSES AREAL MATRIX BY GAUSSIAN ELIMINATION,
C  AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C  IN1UT..
C
C     NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING   A
C     N    = ORDER OF THE MATRIX
C     A    = MATRIX TO BE TRIANGULARIZED
C
C  OUTPUT..
C
C     A   CONTAINS AN UPPER TRIANGULAR MATRIX  U  AND A PREMUTED
C         VERSION OF A LOWER TRIANGULAR MATRIX  I-L  SO THAT
C         (PERMUTATION MATRIX)*A=L*U
C
C     COND = AN ESTIMATE OF THE CONDITION OF  A.
C       FOR THE LINEAR SYSTEM  A*X = B , CHANGES IN   A  AND  B
C       MAY CAUSE CHANGES  COND  TIMES AS LARGE IN  X.
C       IF  COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C       PRECISION.  COND  IS SET TO  1.0D+32  IF EXACT
C       SINGULARITY IS DETECTED.
C
C     IPVT      = THE PIVOT VECTOR
C       IPVT(K) = THE INDEX OF THE K-TH PIVOT ROW
C       IPVT(N) = (-1)**(NUMBER OF INTERCHANGES)
C
C  WORK SPACE..  THE VECTOR  WORK  MUST BE DECLARED AND INCLUDED
C       IN THE CALL.  ITS IN1UT CONTENTS ARE IGNORED.
C       ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
C
C  THE DETERMINANT OF  A  CAN BE OBTAINED ON OUTPUT BY
C     DET(A) = IPVT(N) * A(1,1) * A(2,2) * ... * A(N,N) .
C
      IPVT(N)=1
      IF (N.EQ.1) GO TO 150
      NM1=N-1
C                             COMPUTE THE 1-NORM OF  A .
      ANORM=0.D0
      DO 20 J=1,N
        T=0.D0
        DO 10 I=1,N
   10   T=T+DABS(A(I,J))
        IF (T.GT.ANORM) ANORM=T
   20 CONTINUE
C                             DO GAUSSIAN ELIMINATION WITH PARTIAL
C                                  PIVOTING.
      DO 70 K=1,NM1
```

```
      KP1=K+1
C                             FIND THE PIVOT.
      M=K
      DO 30 I=KP1,N
        IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
   30 CONTINUE
      IPVT(K)=M
      IF (M.NE.K) IPVT(N)=-IPVT(N)
      T=A(M,K)
      A(M,K)=A(K,K)
      A(K,K)=T
C                             SAIP THE ELIMINATION STEP IF PIVOT IS ZERO.
      IF (T.EQ.0.D0) GO TO 70
C
C                             COMPUTE THE MULTIPLIERS.
      DO 40 I=KP1,N
   40 A(I,K)=-A(I,K)/T
C                             INTERCHANGE AND ELIMINATE BY COLUMNS.
      DO 60 J=KP1,N
        T=A(M,J)
        A(M,J)=A(K,J)
        A(K,J)=T
        IF (T.EQ.0.D0) GO TO 60
        DO 50 I=KP1,N
   50   A(I,J)=A(I,J)+A(I,K)*T
   60 CONTINUE
   70 CONTINUE
C
C  COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C  THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C  SMALL SINGULAR VECTOR.  THIS INVOLVES SOLVING TWO SYSTEMS
C  OF EQUATIONS,  (A-TRANSPOSE)*Y = E  AND  A*Z = Y  WHERE  E
C  IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C  ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C
C                             SOLVE  (A-TRANSPOSE)*Y = E .
      DO 100 K=1,N
        T=0.D0
        IF (K.EQ.1) GO TO 90
        KM1=K-1
        DO 80 I=1,KM1
   80   T=T+A(I,K)*WORK(I)
   90   EK=1.D0
        IF (T.LT.0.D0) EK=-1.D0
        IF (A(K,K).EQ.0.D0) GO TO 160
  100 WORK(K)=-(EK+T)/A(K,K)
      DO 120 KB=1,NM1
        K=N-KB
        T=0.D0
        KP1=K+1
        DO 110 I=KP1,N
  110   T=T+A(I,K)*WORK(K)
        WORK(K)=T
        M=IPVT(K)
        IF (M.EQ.K) GO TO 120
        T=WORK(M)
        WORK(M)=WORK(K)
        WORK(K)=T
```

```
  120 CONTINUE
C
      YNORM=0.D0
      DO 130 I=1,N
  130 YNORM=YNORM+DABS(WORK(I))
C
C                                SOLVE  A*Z = Y
      CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
      ZNORM=0.D0
      DO 140 I=1,N
  140 ZNORM=ZNORM+DABS(WORK(I))
C
C                                ESTIMATE THE CONDITION.
      COND=ANORM*ZNORM/YNORM
      IF (COND.LT.1.D0) COND=1.D0
      RETURN
C                                1-BY-1 CASE..
  150 COND=1.D0
      IF (A(1,1).NE.0.D0) RETURN
C
C                                EXACT SINGULARITY
  160 COND=1.0D32
      RETURN
      END
      SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C  SOLVES A LINEAR SYSTEM,  A*X = B
C  DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  IN1UT..
C
C     NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING  A
C     N    = ORDER OF MATRIX
C     A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C     B    = RIGHT HAND SIDE VECTOR
C     IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C  OUTPUT..
C
C     B = SOLUTION VECTOR,  X
C
C                                DO THE FORWARD ELIMINATION.
      IF (N.EQ.1) GO TO 50
      NM1=N-1
      DO 20 K=1,NM1
        KP1=K+1
        M=IPVT(K)
        T=B(M)
        B(M)=B(K)
        B(K)=T
        DO 10 I=KP1,N
```

```
   10    B(I)=B(I)+A(I,K)*T
   20 CONTINUE
C                                      NOW DO THE BACA SUBSTITUTION.
      DO 40 KB=1,NM1
        KM1=N-KB
        K=KM1+1
        B(K)=B(K)/A(K,K)
        T=-B(K)
        DO 30 I=1,KM1
   30    B(I)=B(I)+A(I,K)*T
   40 CONTINUE
   50 B(1)=B(1)/A(1,1)
      RETURN
      END
```

FLOWCHART FOR PROGRAM II

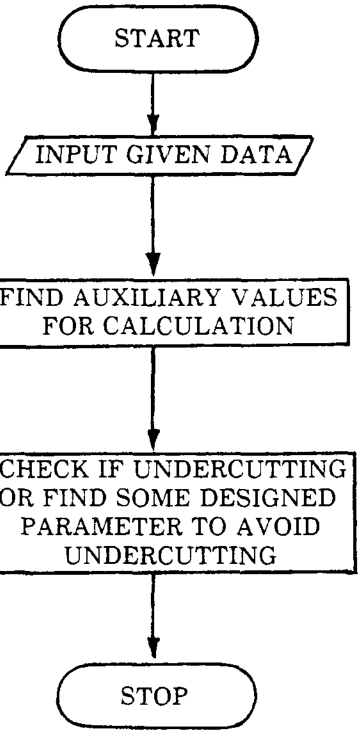


```
      ALP=80.D0*DR
      RC=1.D0
C   (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C        NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM 0. TO
C        (NG-1)*GAMMAI); GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
      N=500
      NG=2
      GAMMAI=5.D0
C   (4) OUTPUT: ZI=INCREMENT OF TOOTH LENTGH OF CROSS SECTION;
C        NL=NO. OF CROSS SECTIONS WHERE PINION PROFILE IS SIMULATED;
C        N=NO. OF POINTS USED TO DEVIDE TOOL PROFILE IN ANY CROSS
C        SECTION(LARGE N, MORE POINTS IS GOT FOR PINION TOOTH PROFILE)
      ZI=0.10D0
      NL=3
      N=500
C
C  DESCRIPTION OF OUTPUT PARAMETERS
C
C     Z1=DISTANCE BETWEEN CROSS SECTION CONSIDERED AND MIDDLE CROSS
C        SECTION
C     NO=OUTPUT NO.
C     Y1=TOOL SURFACE AUXILIARY VARIABLE
C     XP=X COORDINATE OF PINION PROFILE
C     YP=Y COORDINATE OF PINION PROFILE
C     R1=RADIUS OF PINION PROFILE
C     FEE=CORESPONDING PINION SURFACE PARAMETER
C     XSH=AVERAGE DEVIATION SHIFT OF CROSS SECTION PROFILE FROM PROFILE
C         OF MIDDLE SECTION(INVOLUTE CURVE)
C     XPE=MAXIMUM DEVIATION OF CROSS SECTION PROFILE FROM INVOLUTE CURVE
C     SDX=STANDARD DEVIATION OF CROSS SECTION PROFILE FROM INVOLUTE
C         CURVE
C
C  FIND AUXILIARY VALUES FOR CALCULATION
      RP=FLOAT(N1)/2.D0/PN
      CL=RC/DSIN(ALP)
      D=CL*DCOS(ALP)
      A1=CL-HDC/DCOS(RKS)
      RPU=RP+HAC
      DO 5 I=1,NL
      Z=FLOAT(I-1)*ZI
      YY=D-Z/DTAN(ALP)
      WRITE (LP,10) Z
   10 FORMAT (1H1/1X,'Z1=',F15.7/1X,'NO',10X,'Y1',13X,'XP',13X,'YP',
     +        13X,'R1',13X,'FEE')
C  CALCULATE THE PROPILE OF THE SURFACE CUT BY PLANE Z=CONST
      KKK=0
      DO 15 J=1,N
      Y1=YY*FLOAT(J-1)/FLOAT(N-1)
      F=DSQRT(1.-(Z/(D-Y1)/DTAN(ALP))**2)
      FEE=((D-Y1)*F/DCOS(ALP)-A1*(F*DCOS(ALP)*DCOS(ALP)+DSIN(ALP)*DSIN(
     +   ALP)))/RP/(DSIN(ALP)*DCOS(ALP-RKS)-F*DCOS(ALP)*DSIN(ALP-RKS))
      XP(I,J)=(D-Y1)*(DTAN(ALP)*F*DCOS(ALP-RKS+FEE)-DSIN(ALP-RKS+FEE))
     +   +A1*DSIN(FEE-RKS)-RP*FEE*DCOS(FEE)+RP*DSIN(FEE)
      YP(I,J)=-(D-Y1)*(DTAN(ALP)*F*DSIN(ALP-RKS+FEE)+DCOS(ALP-RKS+FEE))
     +   +A1*DCOS(FEE-RKS)+RP*FEE*DSIN(FEE)+RP*DCOS(FEE)
      R1=DSQRT(XP(I,J)**2+YP(I,J)**2)
      IF (R1.GT.RPU) THEN
      JJ=J-1
```

```
C... *******************************************************
C... *                                                     *
c... *                                                     *
C... *                      PROGRAM II                     *
C... *    SURFACE OF PINION GENERRATED BY CONE CUTTER      *
C... *                                                     *
C... *              AUTHORS: FAYDOR LITVIN                 *
C... *                       JIAO ZHANG                    *
C... *                                                     *
C... *                                                     *
C... *******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE SURFACE OF A PINION WHICH IS
C     GENERATED BY CONE CUTTER
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION NS(15),XP(15,999),YP(15,999),XERROR(999)
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C         EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       RKSTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C       NC, EPSI AND RKSTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NC=100
      RKSTA=1.D-3
      EPSI=1.D-12
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C        RKS=PRESSURE ANGLE(DEGREE); HDC=HEIGHT OF DEDENDUM OF PINION;
C        HAC=HEIRHT OF ADDENDUM OF PINION
      PN=10.D0
      N1=20
      RKS=20.D0*DR
      HDC=1.D0/PN
      HAC=1.D0/PN
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C        RC=RADIUS OF BOTTOM CIRCLE OF CONE
```

```
      XP(I,J)=XP(I,JJ)+(XP(I,J)-XP(I,JJ))/(R1-R1TEMP)*(RPU-R1TEMP)
      YP(I,J)=YP(I,JJ)+(YP(I,J)-YP(I,JJ))/(R1-R1TEMP)*(RPU-R1TEMP)
      FEE=FEETEM+(FEE-FEETEM)/(R1-R1TEMP)*(RPU-R1TEMP)
      Y1=Y1TEM+(Y1-Y1TEM)/(R1-R1TEMP)*(RPU-R1TEMP)
      R1=DSQRT(XP(I,J)**2+YP(I,J)**2)
      KKK=1
      END IF
      WRITE (LP,20) J,Y1,XP(I,J),YP(I,J),R1,FEE
   20 FORMAT (1X,I4,4F15.7,F15.7)
      IF (KKK.GT.0) GO TO 30
      FEETEM=FEE
      Y1TEM=Y1
      R1TEMP=R1
   15 CONTINUE
   30 NS(I)=J-1
      IF (I.NE.1) GO TO 55
      NS1=NS(1)
      GO TO 5
C PREPARATION OF INTERPLORATION
   55 NS2=NS(I)
      DO 105 L=1,NS2
      J=NS2+1-L
      IF (YP(1,NS1).GT.YP(I,J)) GO TO 110
  105 CONTINUE
  110 NS2=J
      NREC=2
      XERS=0.D0
      DO 115 L=1,NS2
      DO 125 J=NREC,NS1
      IF (YP(1,J).GT.YP(I,L)) GO TO 120
  125 CONTINUE
  120 NREC=J
      J1=J-1
      XERROR(L)=XP(I,L)-XP(1,J1)-(YP(I,L)-YP(1,J1))*(XP(1,J)-XP(1,J1))
     +          /(YP(1,J)-YP(1,J1))
      XERS=XERS+XERROR(L)
  115 CONTINUE
      XERS=-XERS/FLOAT(NS2)
      XPE=0.D0
      SDX=0.D0
      IF (NDBUG.GT.2) WRITE (LP,80)
   80 FORMAT (1H1,13X,'NO.',4X,'DIVIATION VALUE')
      DO 45 L=1,NS2
      XERROR(L)=XERROR(L)+XERS
      IF (NDBUG.GT.2) WRITE (LP,50) L,XERROR(L)
   50 FORMAT (13X,I3,2F15.7)
      IF (DABS(XERROR(L)).GT.XPE) XPE=DABS(XERROR(L))
      SDX=SDX+XERROR(L)**2
   45 CONTINUE
      SDX=DSQRT(SDX/FLOAT(NS2))
      WRITE (LP,40) XERS,XPE,SDX
   40 FORMAT (///1X,'XSH=',E15.7,5X,'XPE=',E15.7,5X,'SDX=',E15.7)
      IF (NDBUG.GT.2) WRITE (LP,60) NS1,NS2
   60 FORMAT (//5X,'NS1=',I6,6X,'NS2=',I6)
    5 CONTINUE
      STOP
      END
```

FLOWCHART FOR PROGRAM III

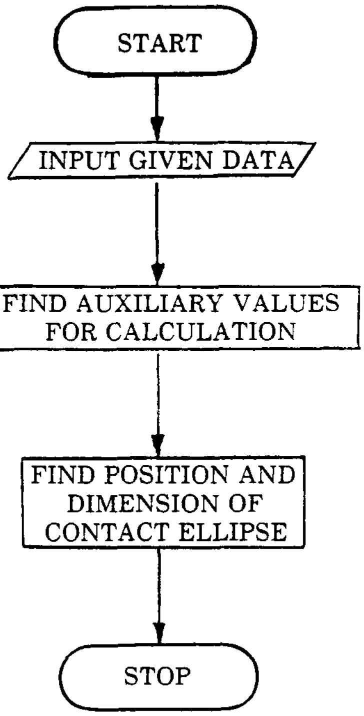


```
C... ******************************************************
C... *                                                    *
C... *                                                    *
C... *                    PROGRAM III                     *
C... *   UDDERCUTTING CONDITION FOR PINION GENERRATED     *
C... *                  BY CONE CUTTER                    *
C... *                                                    *
C... *               AUTHOR: FAYDOR LITVIN                *
C... *                       JIAO ZHANG                   *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO FIND THE UDDERCUTTING CONDITIONS FOR A
C     PINION GENERATED BY CONE CUTTER
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) OTHER PARAMETERS(DON'T CHANGE)
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C        RKSI=PRESSURE ANGLE; HDC=HEIGHT OF DEDENDUM OF PINION;
C        TL=TOOTH LENGTH
      PN=10.D0
      N1=20
      RKSI=20.D0*DR
      HDC=1.D0/PN
      TL=6.D0/PN
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C        RC=RADIUS OF BOTTOM CIRCLE OF CONE
      ALPHA=80.D0*DR
      RC=1.D0
C   (3) PROBLEM: NPROB=ID NO. OF PROBLEM (-1=GIVEN N1 AND HDC, FIND IF
C        UDDERCUTTING OCCUR; 0=GIVEN N1, FIND MAXIMUM HDC WITHOUT UDDER-
C        CUTTING; 1=GIVEN HDC, FIND MINIMUM N1 WITHOUT UDDERCUTTING);
C        N=NO. OF THETA VALUES USED CALCULATION (BETWEEN THETAS CORRES-
C        PONDING TO MIDDLE SECTION AND EDGE SECTION OF PINION)
      NPROB= 1
      N=11
C
```

```
C DESCRIPTION OF OUTPUT
C
C   OUTPUT IS A STATEMENT BASED ON THE PROBLEM WITHOUT ANY LITERAL
C      PARAMETER
C
C  FIND AUXILIARY VALUES FOR CALCULATION
      CL=RC/DSIN(ALPHA)
      SA=DSIN(ALPHA)
      CA=DCOS(ALPHA)
      SK=DSIN(RKSI)
      CK=DCOS(RKSI)
      SAK=DSIN(ALPHA-RKSI)
      CAK=DCOS(ALPHA-RKSI)
      THEMAX=DARSIN(TL/2.0/SA)
      IF (NPROB.NE.0) THEN
      A1=(CL-HDC/CK)*CK
      ACL=A1/CL
      END IF
      IF (NPROB.LT.1) THEN
      RP=FLOAT(N1)/2./PN
      RCL=RP/CL
      END IF
      WRITE (LP,90)
   90 FORMAT (1H1)
      IF (NPROB) 5,15,25
C  CHECK IF UNDERCUTTING OCCURS
    5 IF (NDBUG.LT.1) WRITE (LP,10)
   10 FORMAT (1H1/3X,'NO',7X,'THETA',12X,'A',13X,'B',14X,'C',10X,
     +        'B**2-4.*A*C',3X,'U1/(RC/SIN(ALPHA))')
      UMIN=5.D0
      DO 45 I=1,N
      THE=THEMAX*DBLE(FLOAT(I-1))/DBLE(FLOAT(N-1))
      ST=DSIN(THE)
      CT=DCOS(THE)
      COE1=CA*CAK+CT*SA*SAK
      COE2=SA*CAK-CT*CT*CT*CA*SAK
      COE3=SA*CAK-CT*CA*SAK
      COE4=ST*ST*SA*CA
      A=COE1*COE2
      B=-ACL*(COE2+COE1*COE4)-RCL*COE3**3
      C=ACL*ACL*COE4
      DD=B*B-4.*A*C
      IF (DD.GE.0.) THEN
      U1CL=(-B+DSQRT(DD))/2./A
      IF (UMIN.GT.U1CL) UMIN=U1CL
      U2CL=(-B-DSQRT(DD))/2./A
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD,U1CL
  100 FORMAT (1X,I4,8F15.7)
      ELSE
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD
      END IF
   45 CONTINUE
      IF (UMIN.LT.1.D0) WRITE (LP,110)
  110 FORMAT (///1X,'UDDERCUTTING WILL OCCUR FOR YOUR DESIGN')
      IF (UMIN.GE.1.D0) WRITE (LP,120)
  120 FORMAT (///1X,'UDDERCUTTING WILL NOT OCCUR FOR YOUR DESIGN')
      GO TO 35
C  DETERMINE THE MAXIMUM ADDENDUM HEIGHT OF RACA CUTTER
```

```
   15 IF (NDBUG.LT.1) WRITE (LP,20)
   20 FORMAT (1H1/3X,'NO',7X,'THETA',12X,'A',13X,'B',14X,'C',10X,
     +         'B**2-4.*A*C',3X,'ALLOWED RATIO OF HDC/(1/PN)')
      UMIN=5.D0
      DO 55 I=1,N
      THE=THEMAX*DBLE(FLOAT(I-1))/DBLE(FLOAT(N-1))
      ST=DSIN(THE)
      CT=DCOS(THE)
      COE1=CA*CAK+CT*SA*SAK
      COE2=SA*CAK-CT*CT*CT*CA*SAK
      COE3=SA*CAK-CT*CA*SAK
      COE4=ST*ST*SA*CA
      C=COE1*COE2-RCL*COE3**3
      B=-(COE2+COE1*COE4)
      A=COE4
      DD=B*B-4.*A*C
      IF (DD.GE.0.) THEN
      IF (A.NE.0.) THEN
      U1CL=(-B-DSQRT(DD))/2./A
      U2CL=(-B+DSQRT(DD))/2./A
      ELSE
      U1CL=-C/B
      U2CL=0.
      END IF
      U1CL=(CK-U1CL)*CL*PN
      U2CL=(CK-U2CL)*CL*PN
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD,U1CL
      ELSE
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,A,B,C,DD
      END IF
      IF (UMIN.GT.U1CL) UMIN=U1CL
   55 CONTINUE
      WRITE (LP,200) UMIN
  200 FORMAT (///1X,'TO AVOID UDDERCUTTING, IT IS NECESSARY TO KEEP DEDE
     +NDUM OF PINION <=',F10.7,'/PN')
      GO TO 35
C  DETERMINE THE MINIMUM NO. OF TEETH FOR UNUNDERCUTTING
   25 IF (NDBUG.LT.1) WRITE (LP,30)
   30 FORMAT (1H1/3X,'NO',7X,'THETA',7X,'NO. OF TEETH')
      RNMAX=0.
      DO 65 I=1,N
      THE=THEMAX*DBLE(FLOAT(I-1))/DBLE(FLOAT(N-1))
      ST=DSIN(THE)
      CT=DCOS(THE)
      COE1=CA*CAK+CT*SA*SAK
      COE2=SA*CAK-CT*CT*CT*CA*SAK
      COE3=SA*CAK-CT*CA*SAK
      COE4=ST*ST*SA*CA
      RR=(COE1*COE2-ACL*(COE2+COE1*COE4)+ACL*ACL*COE4)/COE3**3
      RN=2.*RR*PN*CL
      IF (RNMAX.LT.RN) RNMAX=RN
      IF (NDBUG.LT.1) WRITE (LP,100) I,THE,RN
   65 CONTINUE
      WRITE (LP,300) RNMAX
  300 FORMAT (/// 1X,'WITHOUT UDDERCUTTING, MINIMUM TOOTH NO. OF PINION
     +IS:',F11.7)
   35 WRITE (LP,400)
  400 FORMAT(1H1)
```

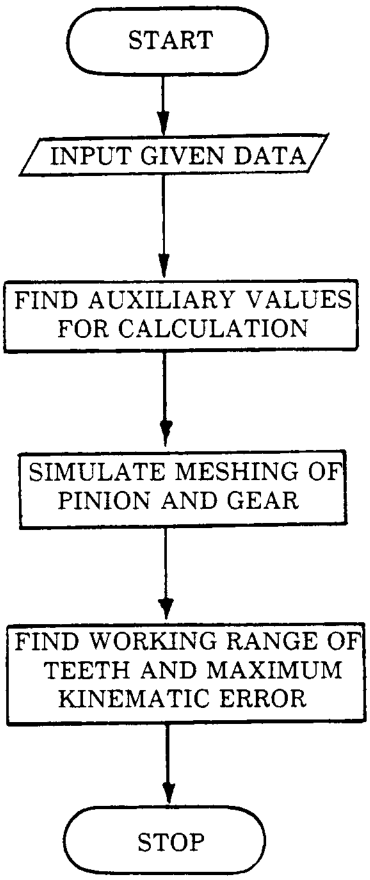
FLOWCHART FOR PROGRAM IV



```
C... ******************************************************
C... *                                                    *
C... *                                                    *
C... *                    PROGRAM IV                      *
C... *  CONTACT ELLIPSIS FOR PINION GENERATED BY CONE      *
C... *      CUTTER IN MESHING WITH REGULAR GEAR           *
C... *                                                    *
C... *            AUTHORS: FAYDOR LITVIN                  *
C... *                     JIAO ZHANG                     *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE CONTACT ELLIPSIS WHEN PINION
C     GENERATED BY CONE CUTTER IN MESHING WITH REGULAR INVOLUTE GEAR
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE IN1UT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) OTHER PARAMETERS(DON'T CHANGE)
      DR=DATAN(1.D0)/45.D0
C
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C         RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C         RKSI=PRESSURE ANGLE
      PN=10.D0
      N1=20
      RMPG=2.D0
      RKSI=20.D0*DR
C   (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C         RC=RADIUS OF BOTTOM CIRCLE OF CONE;
      ALPHA=89.5D0*DR
      RC=1.D0
C   (3) DEFORMATION: DEL=CONTACT DEFORMATION AT CONTACT POINT
      DEL=4.D-4
C   (4) OUTPUT: PD=RECIPROCAL OF INCREMENT OF S VALUE(S IS THE DISTANCE
C         CONTACT POINT AND  INSTANTANEOUS CENTER)
      PD=100.D0
C
C DESCRIPTION OF OUTPUT PARAMETER
C
C   S=DISTANCE BETWEEN CONTACT POIN- AND INSTANTANEOUS CENTER
```

```
C   R1=PINION RADIUS OF CONTACT POINT
C   FEE=PINION ROTATION ANGLE CORRESPONDING TO CONTACT POINT
C   A=LENGTH OF HALF SHORT AXIS OF CONTACT ELLIPSE
C   B=LENTH OF HALF LONG AXIS OF CONTACT ELLIPSE(ALONG DIRECTION OF
C     GEAR TOOTH LEHGTH)
C
C  FIND AUXILIARY VALUES FOR CALCULATION
      RP=FLOAT(N1)/2./PN
      SA=DSIN(ALPHA)
      CA=DCOS(ALPHA)
      SK=DSIN(RKSI)
      CK=DCOS(RKSI)
      RL=RC/SA
      SMAX=(DSQRT(((1.D0+2.D0/FLOAT(N1))/CK)**2-1.D0)-SK/CK)*CK*RP
      NA=IDINT(SMAX*PD+0.5D0)
      SMAX=FLOAT(NA)/PD
      SMIN=(DSQRT(((1.+2./FLOAT(N1)/RMPG)/CK)**2-1.)-SK/CK)*CK*RP*RMPG
      NB=-IDINT(SMIN*PD+0.5D0)
      SMIN=FLOAT(NB)/PD
      N=1+NA-NB
      WRITE (LP,10) RP,N1,RMPG
   10 FORMAT (1H1,///5X,'RADIUS OF THE PITCH CIRCLE OF PINION (RP):',
     +F15.7//5X,'TOOTH NO. OF PINION(N1):',I8//5X,'RATIO OF ANGULAR VELO
     +CITY (OMEGA OF PINION/OMEGA OF GEAR):',F15.7///1X,'OUTPUT DESIGNAT
     +ION: S=DISTANCE BETWEEN CONTACT POINT AND INSTANTANEOUS CENTER'/
     +21X,'R1=PINION RADIUS OF CONTACT POINT; FEE=PINION ROTATION CORRES
     +PONDING TO CONTACT POINT'/21X,'A=LENGTH OF HALF SHORT AXIS; B=LENG
     +TH OF HALF LONG AXIS(ALONG DIRECTION OF GEAR TOOTH LEHGTH)'/////
     +10X,'S',14X,'R1',10X,'FEE1(D)',11X,'A',14X,'B',13X,'B/A'//)
C  FIND POSITION AND DIMENSION OF CONTACT ELLIPSE
      DO 5 L=1,N
      S=SMIN+(SMAX-SMIN)/FLOAT(N-1)*FLOAT(L-1)
      A=DSQRT(DEL*RP*(SK+S/RP)*(RMPG*SK-S/RP)*2.D0/(1.D0+RMPG)/SK)
      B=DSQRT(DEL*2.D0*SA/CA*(RL-2.D0*RP/CK/FLOAT(N1)-S*SK/CK))
      C=B/A
      FEE=S/RP/CK
      R1=RP*DSQRT(1.D0+(FEE*CK)**2+2.D0*FEE*CK*SK)
      FEE=FEE/DR
      WRITE (LP,20) S,R1,FEE,A,B,C
   20 FORMAT(1X,6F15.7)
    5 CONTINUE
      WRITE (LP,30)
   30 FORMAT (1H1)
      STOP
      END
```

FLOWCHART FOR PROGRAM V



```
C... ******************************************************
C... *                                                    *
C... *                                                    *
C... *                    PROGRAM V                       *
C... *  KINEMATIC ERROR OF THE PINION GENERATED BY CONE    *
C... *   CUTTER MESHING WITH A MISALIGNED REGULUAR GEAR    *
C... *                                                    *
C... *             AUTHORS: FAYDOR LITVIN                 *
C... *                      JIAO ZHANG                    *
C... *                                                    *
C... *                                                    *
C... ******************************************************
C
C PURPOSE
C
C   THIS PROGRAM IS USED TO CALCULATE THE KINEMATIC ERROR OF A PINION
C     GENERATEED BY CONE CUTTER MESHING WITH A MISALIGNED REGULAR
C     INVOLUTE GEAR
C
C NOTE
C
C   THIS PROGRAM IS WRITTEN IN FORTRAN 77.  IT CAN BE COMPILED BY V
C     COMPILER IN IBM MAINFRAME OR FORTRAN COMPILER IN VAX SYSTEM.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Z(99),ERR(99),ERROR(99)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,
     +                C,S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R
C
C  DEFINE PARAMETERS USED BY PROGRAMS
C
C   (1) IN ANG LP ARE UNIT NUMBERS ASSIGNED TO THE INPUT AND OUTPUT
C         DEVICES
      IN=5
      LP=6
C   (2) NDBUG IS USE TO CONTROLL THE AUXILIARY OUTPUT FOR DEBUGGING
      NDBUG=2
C   (3) NC IS THE UPPER LIMITATION OF REPEATATION FOR SOLVING NONLINEAR
C         EQUTIONS;
C       EPSI IS THE CLEARANCE OF FUNCTION VALUES WHEN THE FUNCTIONS
C         IS CONSIDERED AS SOLVED (ALL FUNTIONS HAVE FORMS OF F(X)=0);
C       DELTA IS THE RELATIVE DIFFERENCE FOR TAKING DERIVATIVES
C       NC, EPSI AND DELTA MAY BE CHANGED WHEN SOLUTIONS ARE DIVERGENT
C         OR LESS ACCURATE
      NC=100
      DELTA=1.D-3
      EPSI=1.D-12
C   (4) OTHER PARAMETERS(DON'T CHANGE)
      NDIM=10
      NE=5
      DR=DATAN(1.D0)/45.D0
C  DEFINE INPUT PARAMTERS OF PROBLEM(USE INCH AS UNIT OF LENGTH)
C   (1) PINION AND GEAR: PN=DIAMETRAL PITCH; N1=PINION TOOTH NUMBER;
C         RMPG=TOOTH NUMBER RATIO(GEAR TOOTH NO./N1);
C         RKSI=PRESSURE ANGLE; HDC=HEIGHT OF DEDENDUM OF PINION;
C         COE=COEFF. OF CENTRAL DISTANCE(USUALLY COE=1.)
```

```
      PN=10.D0
      N1=20
      RMPG=2.D0
      RKSI=20.D0*DR
      HDC=1.D0/PN
      COE=1.005D0
C    (2) TOOL: ALPHA=HALF OF CONE VERTEX ANGLE(DEGREE);
C         RC=RADIUS OF BOTTOM CIRCLE OF CONE
C         R=RADIUS OF ARC(FOR EXACT CONE CHOOSE R >1.D6*N1/2/PN)
      ALPHA=80.D0*DR
      RC=1.D0
      R=1.0D3
C    (3) MISALIGNMENT: NMIS=ID NO.(1=CROSSING AXES, 2=INTERSECTING AXES);
C         NG=NO. OF MISALIGNED ANGLES TO BE SIMULATED (FROM 0. TO
C         (NG-1)*GAMMAI); GAMMAI=INCREMENT OF MISALIGNED ANGLE(MINUTE);
      NMIS=1
      NG=2
      GAMMAI=5.D0
C    (4) OUTPUT: FEEI=INCREMENT OF ROTATION ANGLE OF PINION(DEGREEE)
      FEEI=1.0D0*DR
C
C   DESCRIPTION OF OUTPUT PARAMERTERS
C
C     FEE1=ROTATION ANGLE OF PINION
C     FEE2=ROTATION ANGLE OF GEAR
C     RP=RADIUS OF PINION CONTACT POINT
C     RG=RADIUS OF GEAR CONTACT POINT
C
C   FIND AUXILIARY VALUES FOR CALCULATION
      RP=FLOAT(N1)/2./PN
      RG=RP*RMPG
      C=(RP+RG)*COE
      CA=DCOS(ALPHA)
      SA=DSIN(ALPHA)
      CK=DCOS(RKSI)
      SK=DSIN(RKSI)
      SAK=DSIN(ALPHA-RKSI)
      CAK=DCOS(ALPHA-RKSI)
      RL=RC/SA
      A1=RL-HDC/CK
      NCOEF=360.D0*DR/FEEI/FLOAT(N1)+0.3D0
      N=NCOEF*2+1
      NT=N-NCOEF
      FII=360.D0/RMPG/FLOAT(N1)
C   DEFINE MATRIX DESCIBEING MISALIGNMENT AND OUTPUT ASSEMBLING CONDITION
      DO 5 I=1,3
      DO 5 J=1,3
      S(I,J)=0.D0
      IF (I.EQ.J) S(I,J)=1.D0
    5 CONTINUE
      DO 15 LL=1,NG
      GAMMA=GAMMAI*FLOAT(LL-0)/60.D0*DR
      CG=DCOS(GAMMA)
      SG=DSIN(GAMMA)
      GAMMA=GAMMA/DR*60.D0
      IF (NMIS.EQ.1) THEN
      WRITE (LP,10) COE,GAMMA
   10 FORMAT (1H1,///,1X,'C=',F5.3,'*(RP+RG);     CROSSING ANGLE=',
```


C-7

```
     +        F5.1,'(M)')
      S(1,1)=CG
      S(1,3)=-SG
      S(3,1)=SG
      S(3,3)=CG
      ELSE
      WRITE (LP,20) COE,GAMMA
   20 FORMAT (1H1,///,1X,'C=',F4.2,'*(RP+RG);     INTERSECTING ANGLE=',
     +        F5.1,'(M)')
      S(2,2)=CG
      S(2,3)=-SG
      S(3,2)=SG
      S(3,3)=CG
      END IF
C  SIMULATING MESHING OF PINION AND GEAR(L=1 FOR FINDING INITIAL
C    ROTATION CORRESPONDING TO 0 PINION ROTATION) AND OUTPUT RESULTS
      DO 25 L=1,2
      DO 35 I=1,4
   35 X(I)=0.D0
      DO 45 I=1,N
      X(7)=FEEI*FLOAT(I-(N+1)/2)
      IF (L.EQ.1) X(7)=0.D0
      X(5)=DARSIN(RP*X(7)*SK/R)
      X(2)=X(7)/RMPG
      SF=DSIN(X(7))
      CF=DCOS(X(7))
      CALL NONLIN
      X(8)=X(2)+X(3)
      IF (L.EQ.1) THEN
      XIN=X(8)
      WRITE (LP,30)
   30 FORMAT (////8X,'FEE1(D)',8X,'FEE2(D)',8X,'K-ERROR(S)',5X,
     +        'RP',13X,'RG'/)
      GO TO 25
      END IF
      X(8)=X(8)-XIN
      X(7)=X(7)/DR
      X(8)=X(8)/DR
      X(9)=(X(8)-X(7)/RMPG)*3600.D0
      Z(I)=X(8)
      ERR(I)=X(9)
      WRITE (LP,40) (X(J),J=7,11)
   40 FORMAT (1X,5F15.7,F15.7)
   45 CONTINUE
      WRITE (LP,50)
   50 FORMAT (//,' FIND THE WORKING RANGE FOR ONE TOOTH:'/)
      DO 55 I=1,NT
      X(7)=FEEI*FLOAT(I-(N+1)/2)/DR/2.D0
      X(8)=X(7)+FII
      KK=I+NCOEF
      ANGLE=Z(KK)-Z(I)
      ERROR(I)=(ANGLE-FII)*3600.D0
      WRITE (LP,60) X(7),X(8),ANGLE
   60 FORMAT (1X,'(',F7.2,'----',F7.2,'):',F15.7,F15.7)
   55 CONTINUE
      DO 65 I=1,NT
      ATEMP2=ERROR(I)
      IF (I.NE.1) THEN
```

```
      IF (ATEMP1.GT.0.D0.AND.ATEMP2.LE.0.D0) GOTO 75
      END IF
      ATEMP1=ATEMP2
   65 CONTINUE
      WRITE (LP,70)
   70 FORMAT (//1X,'MESHING IS DISCONTINOUS')
      GOTO 25
   75 IF (DABS(ATEMP1).LT.DABS(ATEMP2)) I=I-1
      EMAX=0.D0
      EMIN=0.D0
      DO 85 J=1,NCOEF
      KS=I+J-1
      ET=ERR(KS)
      IF (ET.LT.EMIN) EMIN=ET
      IF (ET.GT.EMAX) EMAX=ET
   85 CONTINUE
      ET=EMAX-EMIN
      KK=I+NCOEF
      WRITE (LP,80) Z(I),Z(KK),ET
   80 FORMAT (//1X,'WORKING RANGE FOR THE GEAR TOOTH: ',F7.2,'----',
     +     F7.2/1X,'THE MAXIMUM KINEMATIC ERROR: ',F15.7,'  (S)')
   25 CONTINUE
   15 CONTINUE
      WRITE (LP,90)
   90 FORMAT(1H1)
      STOP
      END
C
      SUBROUTINE FUNC
C
C  THIS SUBROUTINE IS USED TO GENERATE FIVE FUNCTIONS, THAT IS, AT
C    CONTACT POINT, POSITION VECTOR AND NORMAL OF PINION AND GEAR
C     MUST COINCIDE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
      COMMON /BLOCA2/ ALPHA,RKSI,RP,RG,RMPG,RL,SA,CA,DR,CK,SK,HDC,
     +                C,S(3,3),SF,CF,SAK,CAK,A1,XP,YP,ZP,XG,YG,R
      CT=DCOS(X(4))
      ST=DSIN(X(4))
      CFEE=DCOS(X(3))
      SFEE=DSIN(X(3))
      CFEK=DCOS(X(3)+RKSI)
      SFEK=DSIN(X(3)+RKSI)
      CAB=DCOS(ALPHA+X(5))
      SAB=DSIN(ALPHA+X(5))
      CAB2=DCOS(ALPHA+X(5)/2.D0)
      SAB2=DSIN(ALPHA+X(5)/2.D0)
      SB=DSIN(X(5))
      SB2=DSIN(X(5)/2.D0)
      A2=A1*SA*(CT-1)
      X(6)=(A2*SAB-R*SB*CT)/(SAB*CAK-CT*CAB*SAK)/RP
      CFP=DCOS(X(6))
      SFP=DSIN(X(6))
      CAKF=DCOS(ALPHA-RKSI+X(6))
      SAKF=DSIN(ALPHA-RKSI+X(6))
      A3=2.D0*R*SB2
      T3=CAB2*SAKF-CT*SAB2*CAKF
```

```
      XP=A3*(CAB2*SAKF-CT*SAB2*CAKF)+A2*CAKF-RP*X(6)*CFP+RP*SFP
      YP=A3*(CAB2*CAKF+CT*SAB2*SAKF)-A2*SAKF+RP*X(6)*SFP+RP*CFP
      ZP=(A1*SA-A3*SAB2)*ST
      RNXP=CT*CAB*CAKF+SAB*SAKF
      RNYP=-CT*CAB*SAKF+SAB*CAKF
      RNZP=ST*CAB
      XG=RG*SFEE+RG*X(2)*CK*CFEK
      YG=-RG*CFEE+RG*X(2)*CK*SFEK
      ZG=X(1)
      RNXG=CFEK
      RNYG=SFEK
      RNZG=0.D0
C     T1=CF*XP+SF*YP
C     T2=S(1,1)*XG+S(1,2)*YG+S(1,3)*ZG
C     WRITE (6,50) T1,T2,XP,YP,X(6)
C  50 FORMAT (1X,'T1=',F15.7,5X,'T2=',F15.7,2F15.7)
      Y(1)=CF*XP+SF*YP-S(1,1)*XG-S(1,2)*YG-S(1,3)*ZG
      Y(2)=CF*YP-SF*XP-S(2,1)*XG-S(2,2)*YG-S(2,3)*ZG-C
      Y(3)=ZP-S(3,1)*XG-S(3,2)*YG-S(3,3)*ZG
      Y(4)=CF*RNYP-SF*RNXP-S(2,1)*RNXG-S(2,2)*RNYG-S(2,3)*RNZG
      Y(5)=RNZP-S(3,1)*RNXG-S(3,2)*RNYG-S(3,3)*RNZG
      X(10)=DSQRT(XP*XP+YP*YP)
      X(11)=DSQRT(XG*XG+YG*YG)
C     WRITE (6,20) XP,YP,ZP,XG,YG,X(6),A1,A2,A3
C  20 FORMAT (1X,'$$$$',6F15.7)
      RETURN
      END
C
      SUBROUTINE NONLIN
C
C  THIS SUBROUTINE IS USED TO SOLVE NONLINEAR EQUATIONS BY NEWTON-
C    RAPHSON METHOD
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /BLOCA1/ X(11),Y(10),A(10,10),Y1(10),IPVT(10),WORK(10),
     +                EPSI,DELTA,NC,NE,NDIM
C
      DO 5 I=1,NC
      CALL FUNC
C     WRITE (6,10) I,(X(J),Y(J),J=1,5)
C  10 FORMAT(1X,'***',I5/5(1X,2D15.7/))
      DO 15 J=1,NE
      IF (DABS(Y(J)).GT.EPSI) GO TO 25
   15 CONTINUE
      GO TO 105
   25 DO 35 J=1,NE
   35 Y1(J)=Y(J)
      DO 45 J=1,NE
      DIFF=DABS(X(J))*DELTA
      IF (X(J).EQ.0.D0) DIFF=DELTA
      XMAM=X(J)
      X(J)=X(J)-DIFF
      CALL FUNC
      X(J)=XMAM
      DO 55 K=1,NE
   55 A(K,J)=(Y1(K)-Y(K))/DIFF
   45 CONTINUE
      DO 65 J=1,NE
   65 Y(J)=-Y1(J)
```

```
      CALL DECOMP (NDIM,NE,A,COND,IPVT,WORK)
      CALL SOLVE (NDIM,NE,A,Y,IPVT)
      DO 75 J=1,NE
   75 X(J)=X(J)+Y(J)
    5 CONTINUE
  105 RETURN
      END
C
C
C
      SUBROUTINE DECOMP (NDIM,N,A,COND,IPVT,WORK)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),WORK(N),IPVT(N)
C
C  DECOMPOSES AREAL MATRIX BY GAUSSIAN ELIMINATION,
C  AND ESTIMATES THE CONDITION OF THE MATRIX.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  USE SUBROUTINE SOLVE TO COMPUTE SOLUTIONS TO LINEAR SYSTEM.
C
C  IN1UT..
C
C     NDIM = DECLARED ROW DIMENSION OF THE ARRAY CONTAINING   A
C     N    = ORDER OF THE MATRIX
C     A    = MATRIX TO BE TRIANGULARIZED
C
C  OUTPUT..
C
C     A   CONTAINS AN UPPER TRIANGULAR MATRIX  U  AND A PREMUTED
C         VERSION OF A LOWER TRIANGULAR MATRIX  I-L  SO THAT
C         (PERMUTATION MATRIX)*A=L*U
C
C     COND = AN ESTIMATE OF THE CONDITION OF  A.
C       FOR THE LINEAR SYSTEM  A*X = B , CHANGES IN   A  AND  B
C       MAY CAUSE CHANGES  COND  TIMES AS LARGE IN  X.
C       IF  COND+1.0 .EQ. COND , A IS SINGULAR TO WORKING
C       PRECISION.  COND  IS SET TO  1.0D+32  IF EXACT
C       SINGULARITY IS DETECTED.
C
C     IPVT      = THE PIVOT VECTOR
C       IPVT(K) = THE INDEX OF THE K-TH PIVOT ROW
C       IPVT(N) = (-1)**(NUMBER OF INTERCHANGES)
C
C  WORK SPACE..   THE VECTOR  WORK  MUST BE DECLARED AND INCLUDED
C       IN THE CALL.  ITS IN1UT CONTENTS ARE IGNORED.
C       ITS OUTPUT CONTENTS ARE USUALLY UNIMPORTANT.
C
C  THE DETERMINANT OF  A  CAN BE OBTAINED ON OUTPUT BY
C     DET(A) = IPVT(N) * A(1,1) * A(2,2) * ... * A(N,N) .
C
      IPVT(N)=1
      IF (N.EQ.1) GO TO 150
      NM1=N-1
C                             COMPUTE THE 1-NORM OF  A .
      ANORM=0.D0
```

```
      DO 20 J=1,N
        T=0.D0
        DO 10 I=1,N
   10   T=T+DABS(A(I,J))
        IF (T.GT.ANORM) ANORM=T
   20 CONTINUE
C                            DO GAUSSIAN ELIMINATION WITH PARTIAL
C                                 PIVOTING.
      DO 70 K=1,NM1
        KP1=K+1
C                            FIND THE PIVOT.
        M=K
        DO 30 I=KP1,N
          IF (DABS(A(I,K)).GT.DABS(A(M,K))) M=I
   30   CONTINUE
        IPVT(K)=M
        IF (M.NE.K) IPVT(N)=-IPVT(N)
        T=A(M,K)
        A(M,K)=A(K,K)
        A(K,K)=T
C                             SAIP THE ELIMINATION STEP IF PIVOT IS ZERO.
        IF (T.EQ.0.D0) GO TO 70
C
C                             COMPUTE THE MULTIPLIERS.
        DO 40 I=KP1,N
   40   A(I,K)=-A(I,K)/T
C                             INTERCHANGE AND ELIMINATE BY COLUMNS.
        DO 60 J=KP1,N
          T=A(M,J)
          A(M,J)=A(K,J)
          A(K,J)=T
          IF (T.EQ.0.D0) GO TO 60
          DO 50 I=KP1,N
   50     A(I,J)=A(I,J)+A(I,K)*T
   60   CONTINUE
   70 CONTINUE
C
C  COND = (1-NORM OF A)*(AN ESTIMATE OF THE 1-NORM OF A-INVERSE)
C  THE ESTIMATE IS OBTAINED BY ONE STEP OF INVERSE ITERATION FOR THE
C  SMALL SINGULAR VECTOR.  THIS INVOLVES SOLVING TWO SYSTEMS
C  OF EQUATIONS,  (A-TRANSPOSE)*Y = E  AND  A*Z = Y  WHERE  E
C  IS A VECTOR OF +1 OR -1 COMPONENTS CHOSEN TO CAUSS GROWTH IN Y.
C  ESTIMATE = (1-NORM OF Z)/(1-NORM OF Y)
C
C                             SOLVE  (A-TRANSPOSE)*Y = E .
      DO 100 K=1,N
        T=0.D0
        IF (K.EQ.1) GO TO 90
        KM1=K-1
        DO 80 I=1,KM1
   80   T=T+A(I,K)*WORK(I)
   90   EK=1.D0
        IF (T.LT.0.D0) EK=-1.D0
        IF (A(K,K).EQ.0.D0) GO TO 160
  100 WORK(K)=-(EK+T)/A(K,K)
      DO 120 KB=1,NM1
        K=N-KB
        T=0.D0
```

```
      KP1=K+1
      DO 110 I=KP1,N
  110 T=T+A(I,K)*WORK(K)
      WORK(K)=T
      M=IPVT(K)
      IF (M.EQ.K) GO TO 120
      T=WORK(M)
      WORK(M)=WORK(K)
      WORK(K)=T
  120 CONTINUE
C
      YNORM=0.D0
      DO 130 I=1,N
  130 YNORM=YNORM+DABS(WORK(I))
C
C                               SOLVE  A*Z = Y
      CALL SOLVE (NDIM,N,A,WORK,IPVT)
C
      ZNORM=0.D0
      DO 140 I=1,N
  140 ZNORM=ZNORM+DABS(WORK(I))
C
C                               ESTIMATE THE CONDITION.
      COND=ANORM*ZNORM/YNORM
      IF (COND.LT.1.D0) COND=1.D0
      RETURN
C                               1-BY-1 CASE..
  150 COND=1.D0
      IF (A(1,1).NE.0.D0) RETURN
C
C                               EXACT SINGULARITY
  160 COND=1.0D32
      RETURN
      END
      SUBROUTINE SOLVE (NDIM,N,A,B,IPVT)
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(NDIM,N),B(N),IPVT(N)
C
C  SOLVES A LINEAR SYSTEM,  A*X = B
C  DO NOT SOLVE THE SYSTEM IF DECOMP HAS DETECTED SINGULARITY.
C
C  -COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS-, BY G. E. FORSYTHE,
C       M. A. MALCOLM, AND C. B. MOLER (PRENTICE-HALL, 1977)
C
C  IN1UT..
C
C     NDIM = DECLARED ROW DIMENSION OF ARRAY CONTAINING  A
C     N    = ORDER OF MATRIX
C     A    = TRIANGULARIZED MATRIX OBTAINED FROM SUBROUTINE DECOMP
C     B    = RIGHT HAND SIDE VECTOR
C     IPVT = PIVOT VECTOR OBTAINED FROM DECOMP
C
C  OUTPUT..
C
C     B = SOLUTION VECTOR,  X
C
C                               DO THE FORWARD ELIMINATION.
```

```
      IF (N.EQ.1) GO TO 50
      NM1=N-1
      DO 20 K=1,NM1
        KP1=K+1
        M=IPVT(K)
        T=B(M)
        B(M)=B(K)
        B(K)=T
        DO 10 I=KP1,N
   10   B(I)=B(I)+A(I,K)*T
   20 CONTINUE
C                           NOW DO THE BACA SUBSTITUTION.
      DO 40 KB=1,NM1
        KM1=N-KB
        K=KM1+1
        B(K)=B(K)/A(K,K)
        T=-B(K)
        DO 30 I=1,KM1
   30   B(I)=B(I)+A(I,K)*T
   40 CONTINUE
   50 B(1)=B(1)/A(1,1)
      RETURN
      END
```

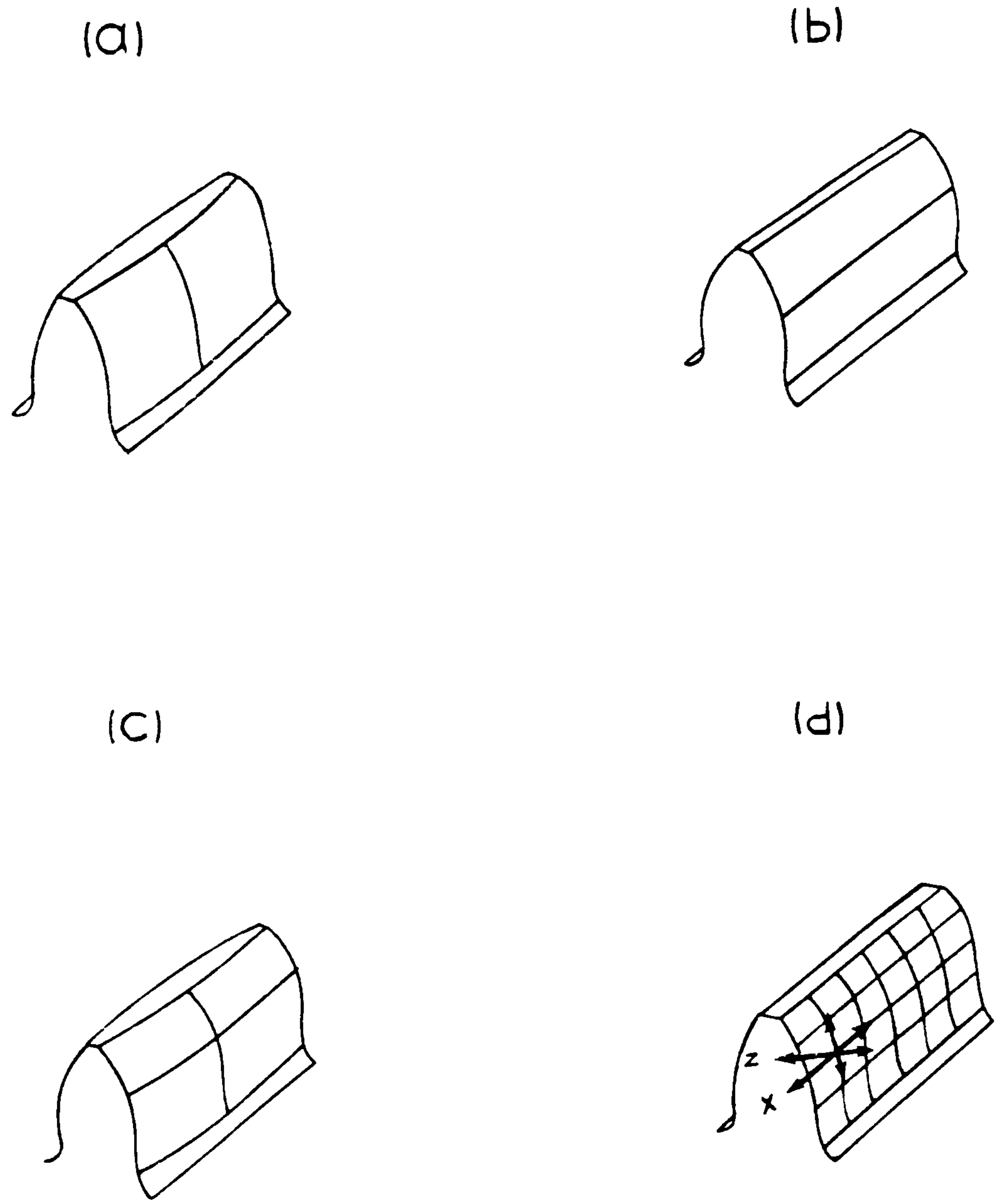
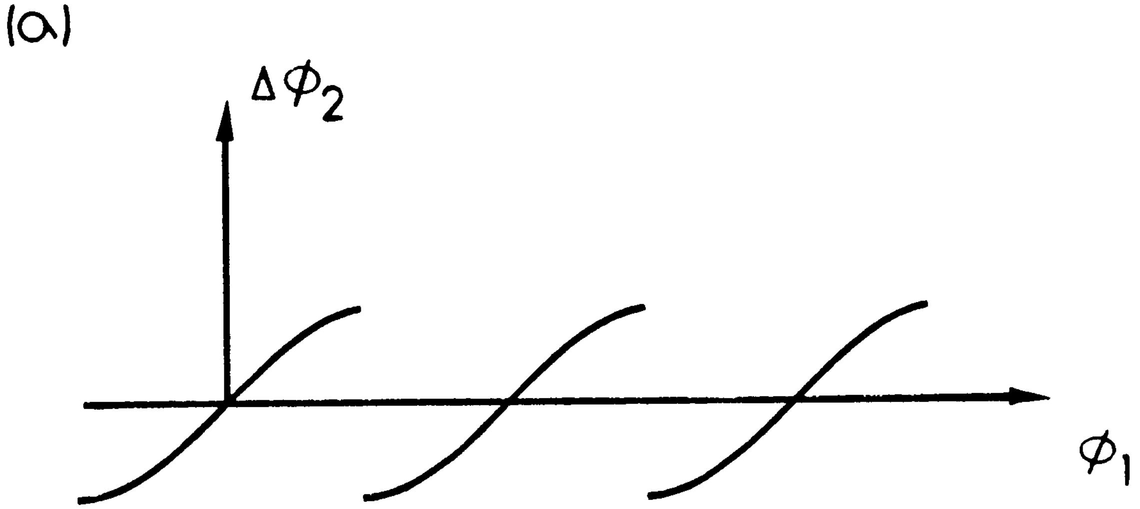


FIG. 1.1



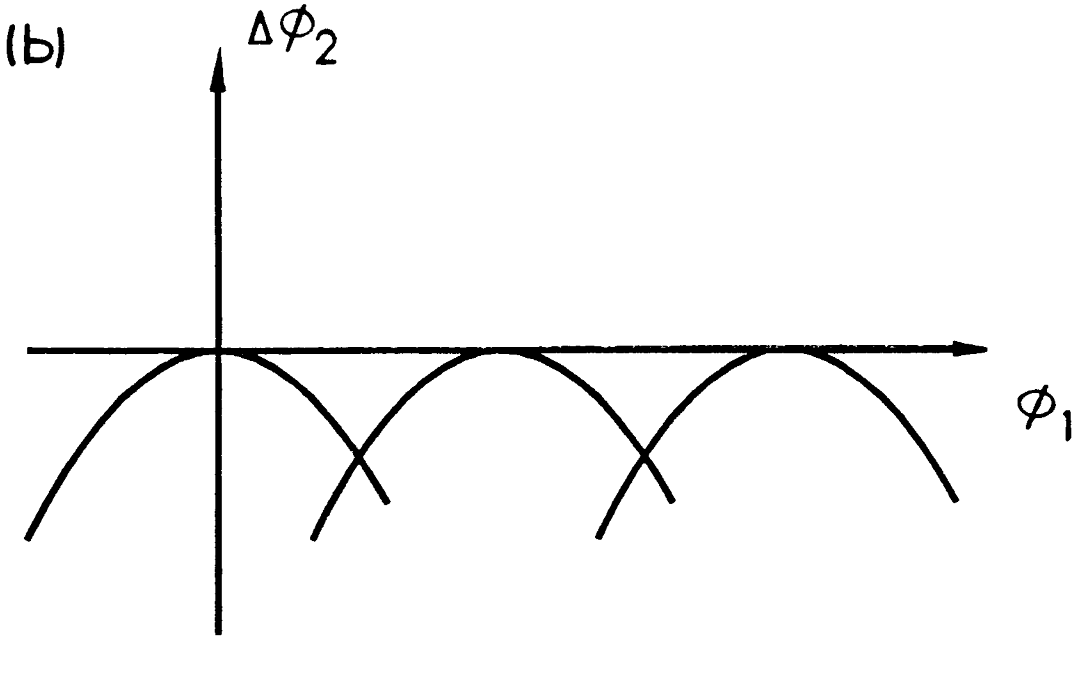


FIG.1.2

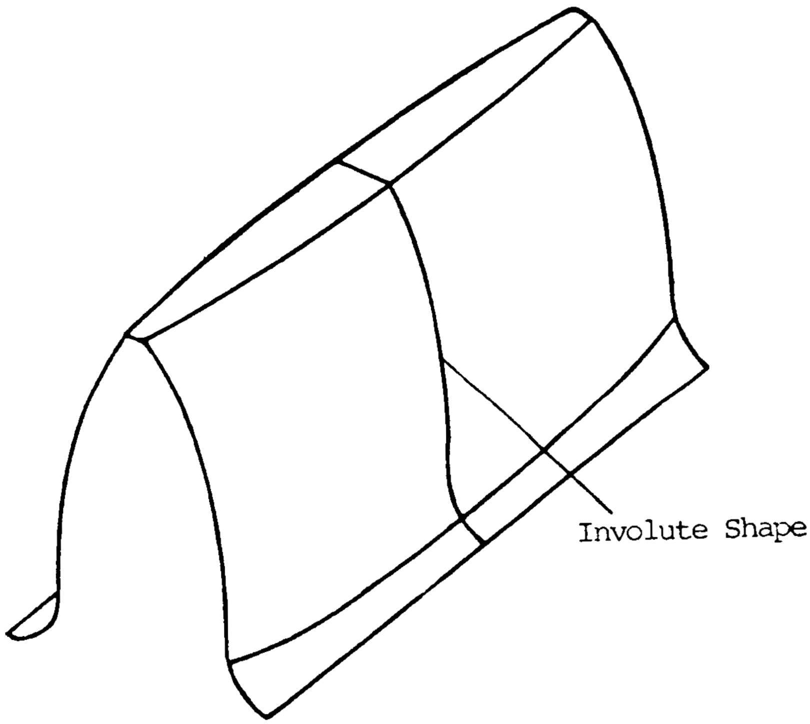


FIG.1.3

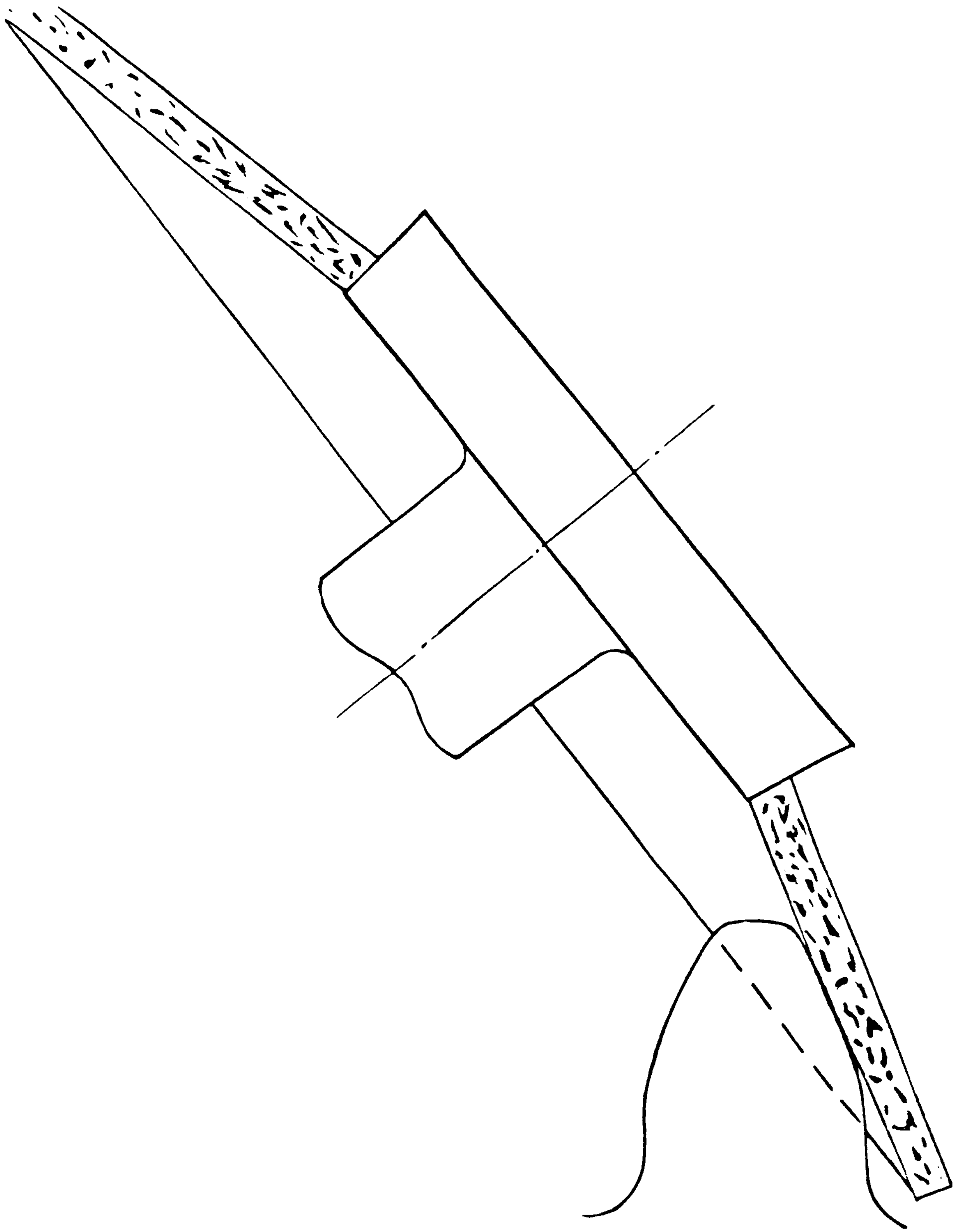
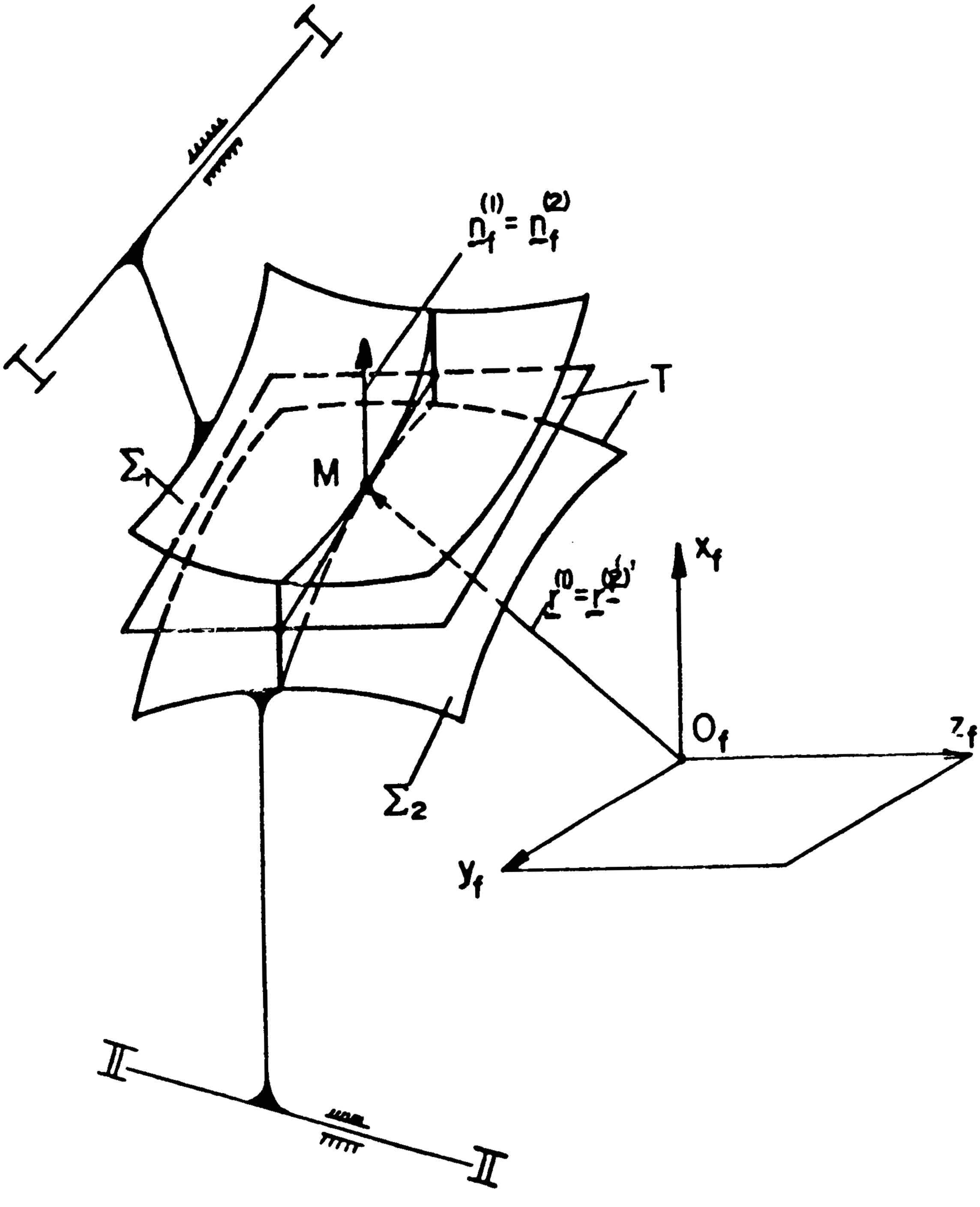
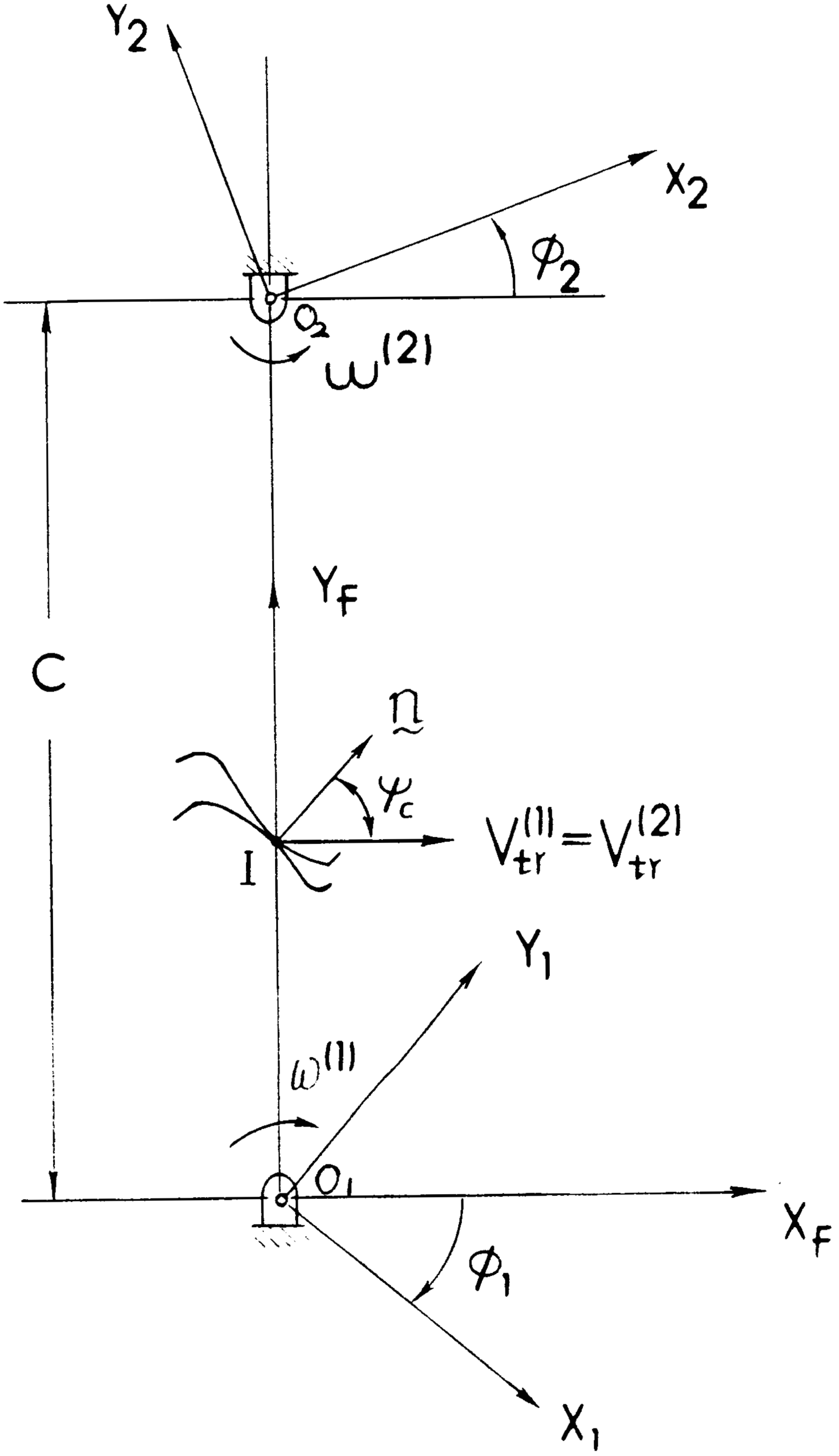


FIG. 1 .4



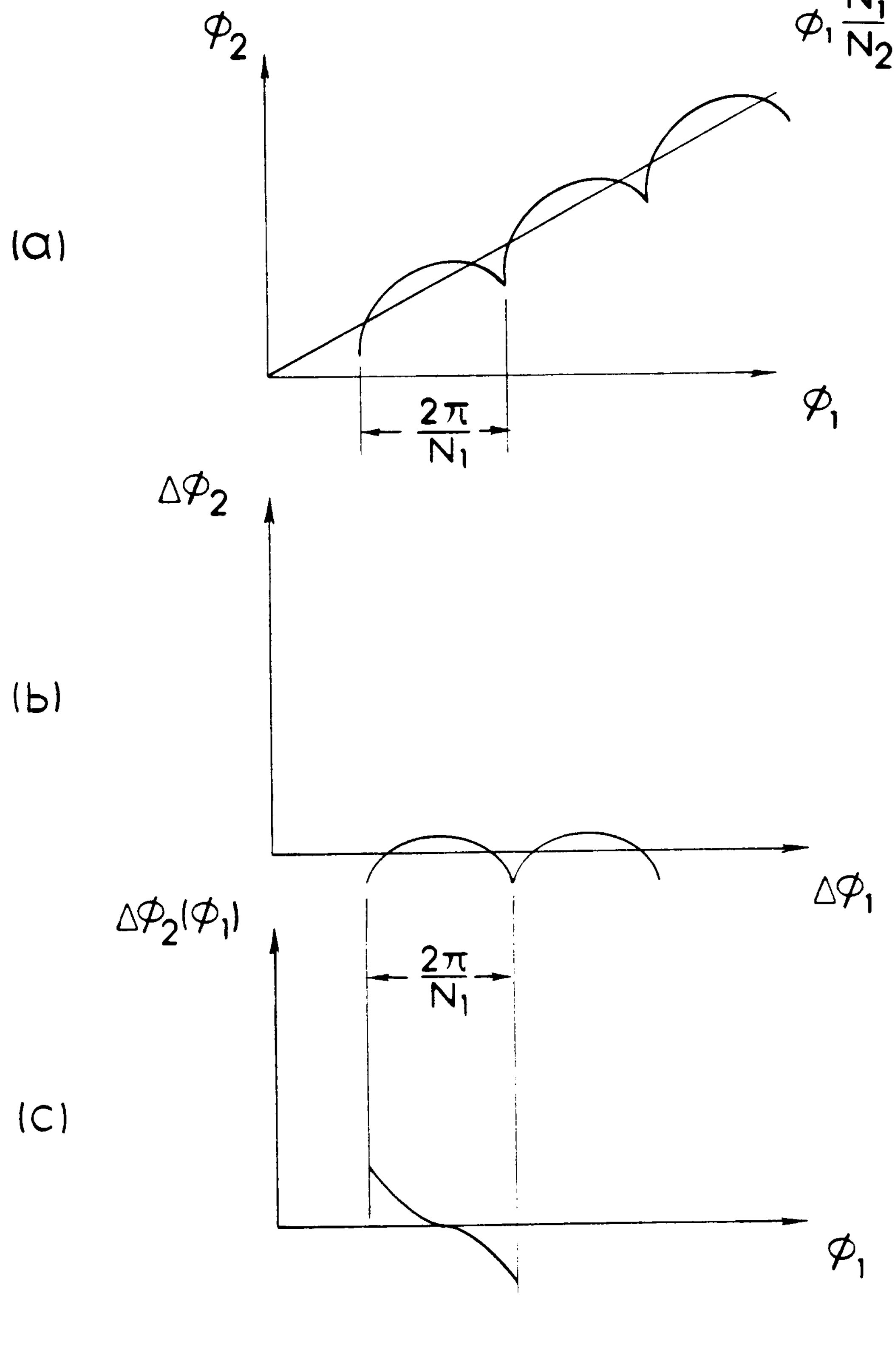
I
I
$\underline{n}_f^{(1)} = \underline{n}_f^{(2)}$
T
Σ_1
M
x_f
$\underline{r}^{(1)} = \underline{r}^{(2)}$
O_f
z_f
Σ_2
y_f
II
II

FIG. 1.5



Y_2
X_2
ϕ_2
O_2
$\omega^{(2)}$
Y_F
C
n
ψ_c
I
$V_{tr}^{(1)} = V_{tr}^{(2)}$
Y_1
$\omega^{(1)}$
O_1
X_F
ϕ_1
X_1

FIG. 2.2.1



ϕ_2
$\phi_1 \frac{N_1}{N_2}$
(a)
ϕ_1
$\frac{2\pi}{N_1}$
$\Delta\phi_2$
(b)
$\Delta\phi_1$
$\Delta\phi_2(\phi_1)$
$\frac{2\pi}{N_1}$
(c)
ϕ_1

FIG. 2.2.2

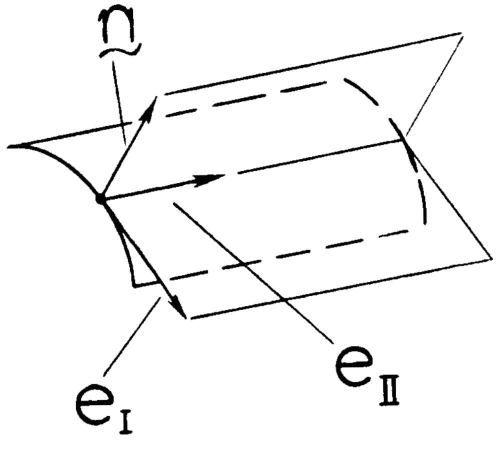


FIG. 2.3.1

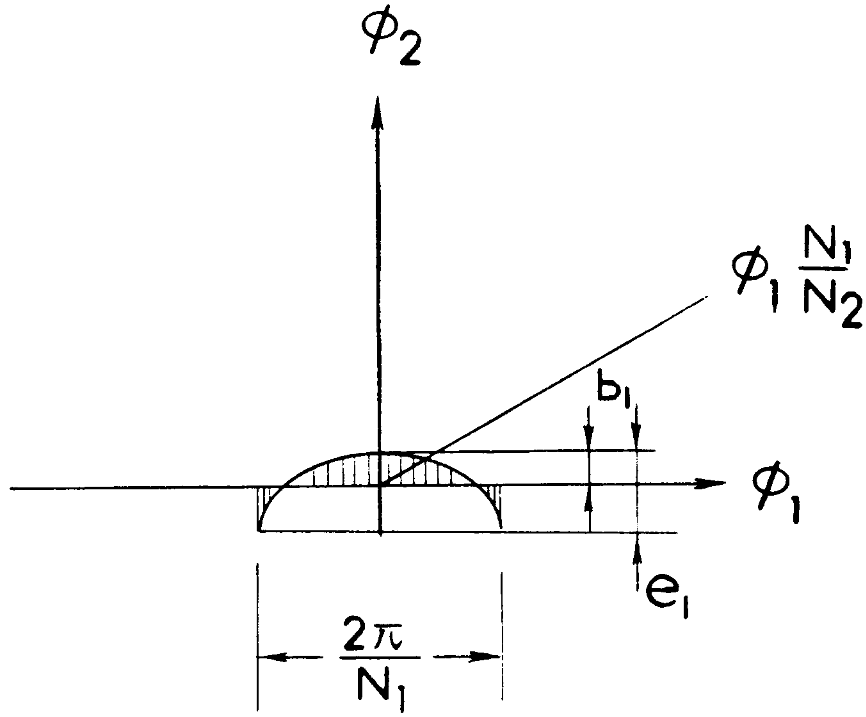


FIG. 3.1.1

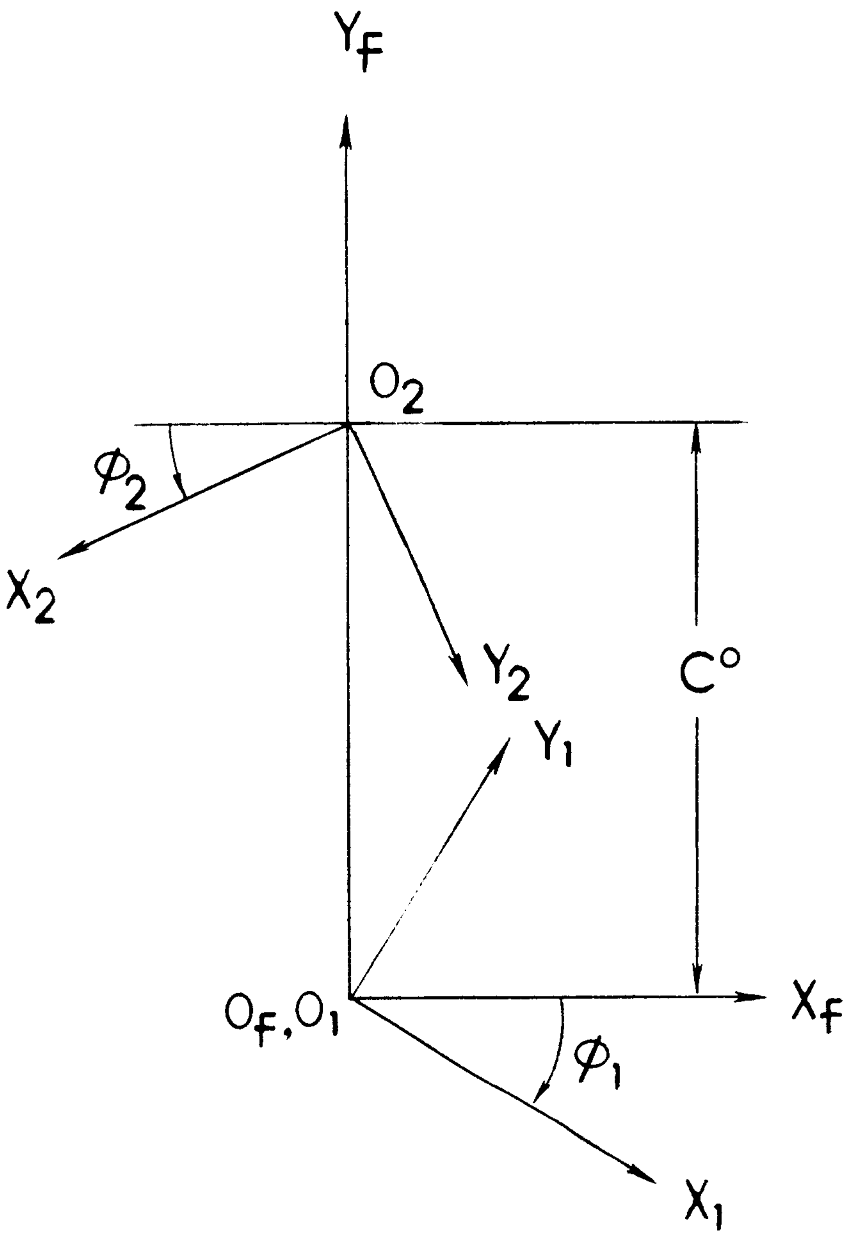


FIG. 3.1.2

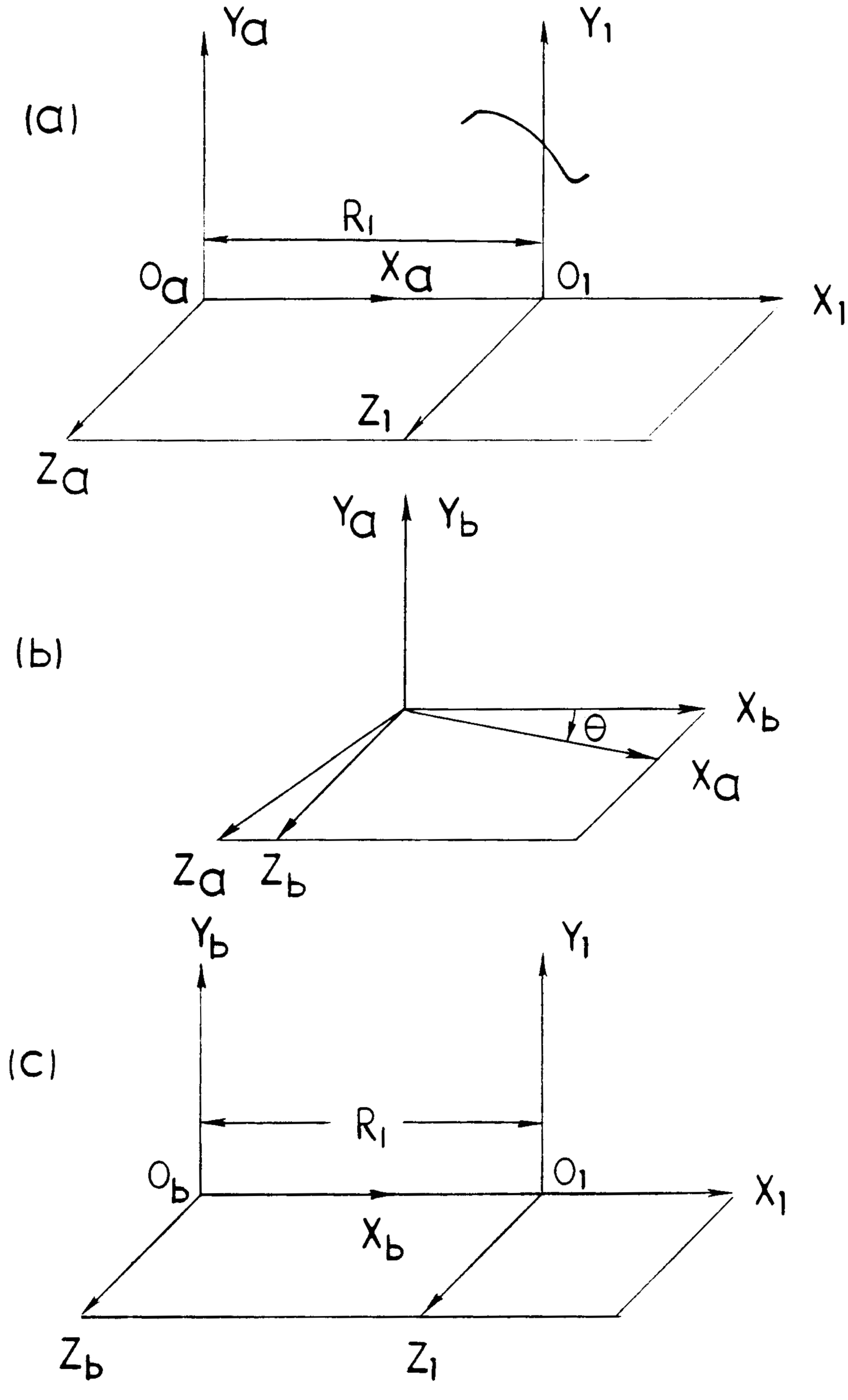

(a)
Ya
Y1
R1
Oa
Xa
O1
X1
Z1
Za
Ya
Yb
(b)
Xb
θ
Xa
Za
Zb
Yb
Y1
(c)
R1
Ob
O1
X1
Xb
Zb
Z1

FIG. 3.2.1

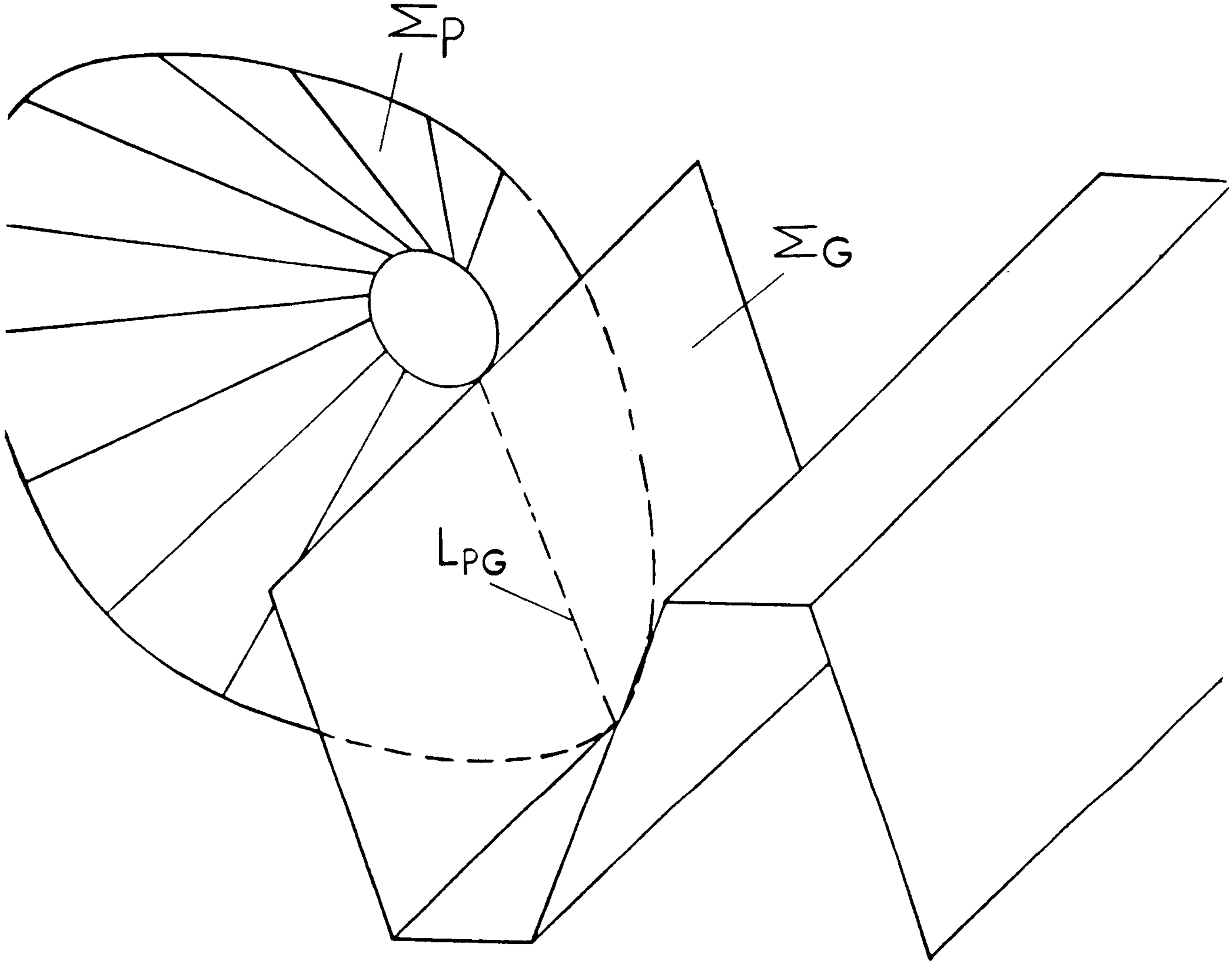


FIG.4.2.0

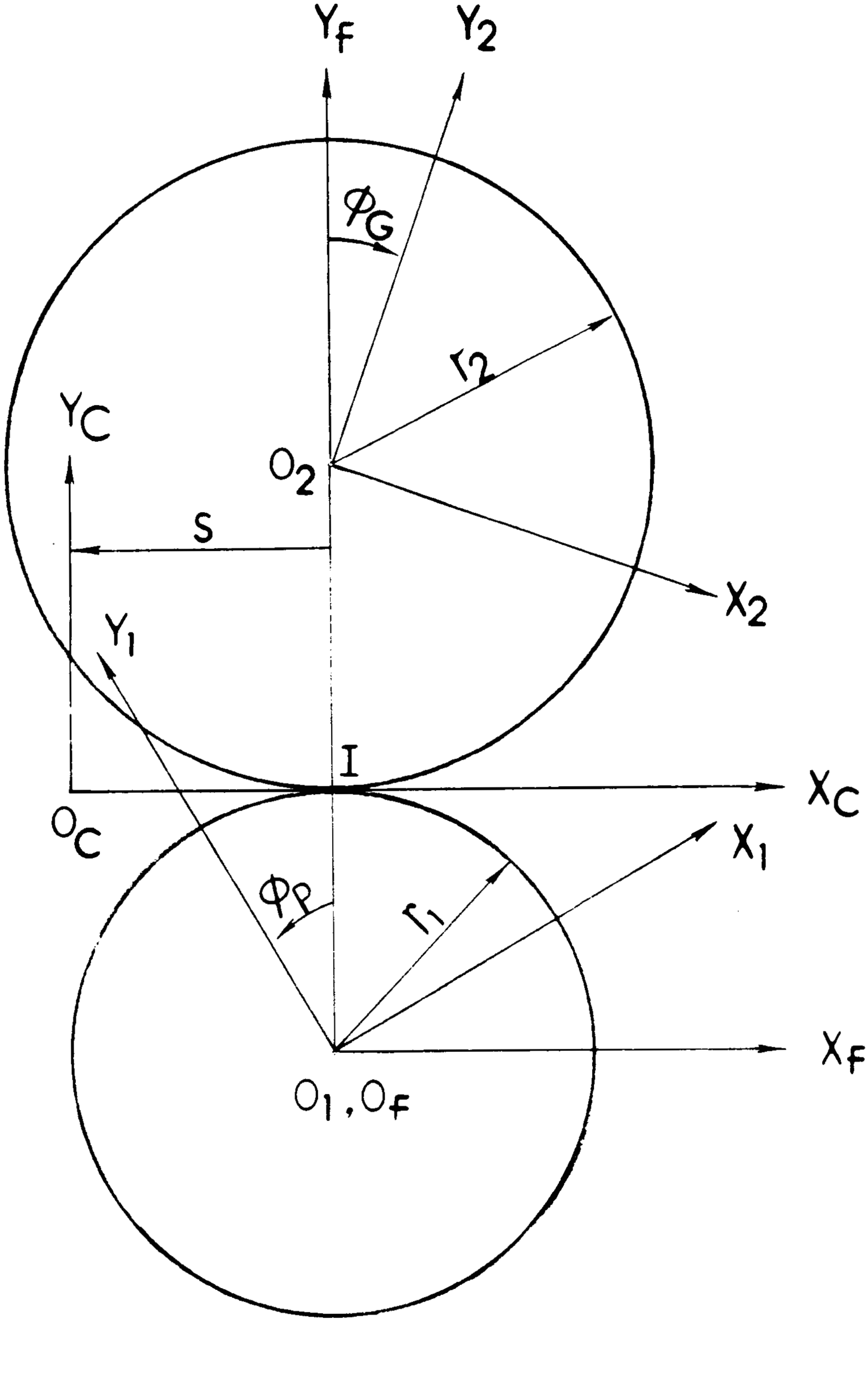


FIG.4.2.1

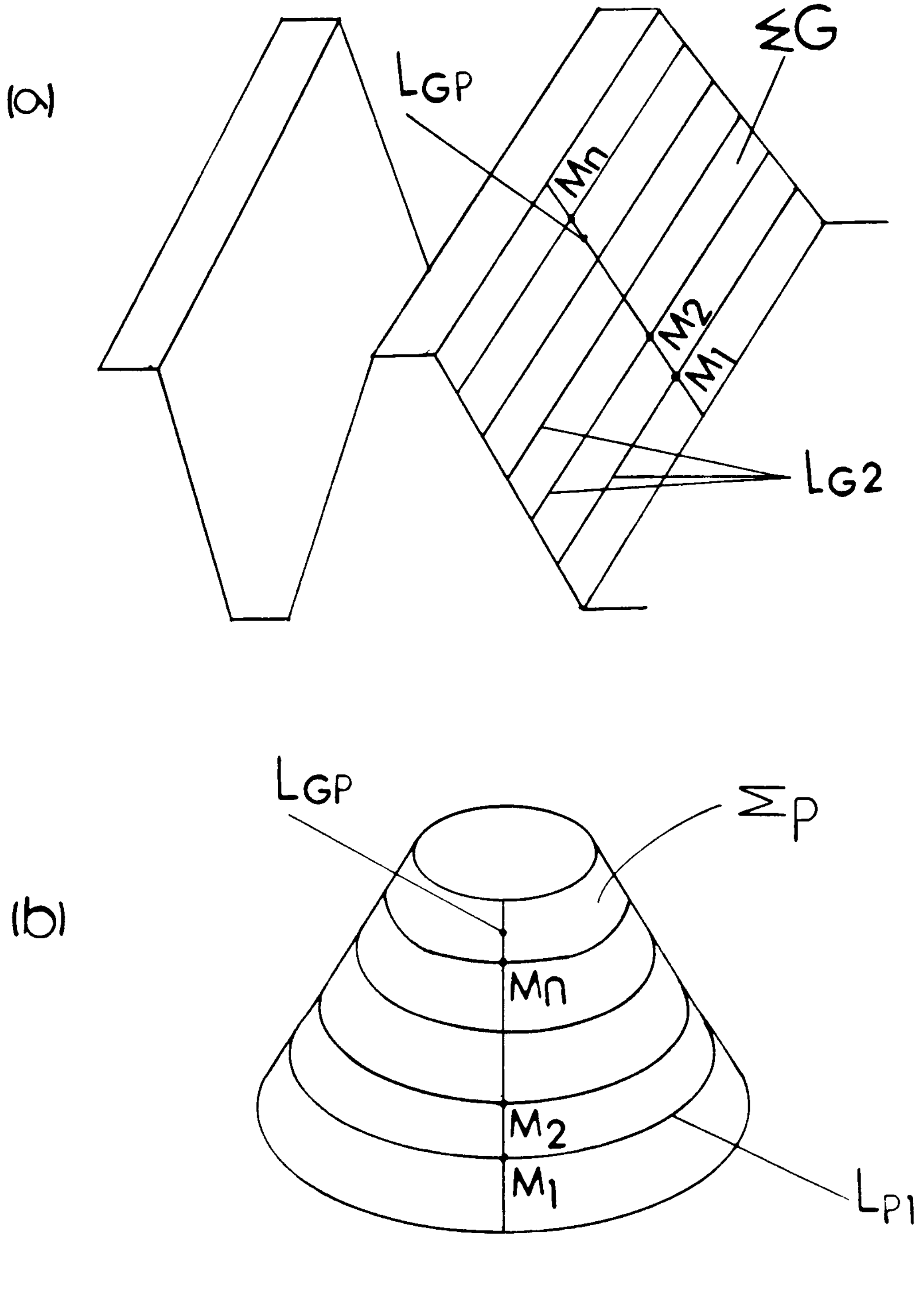


FIG.4.2.2

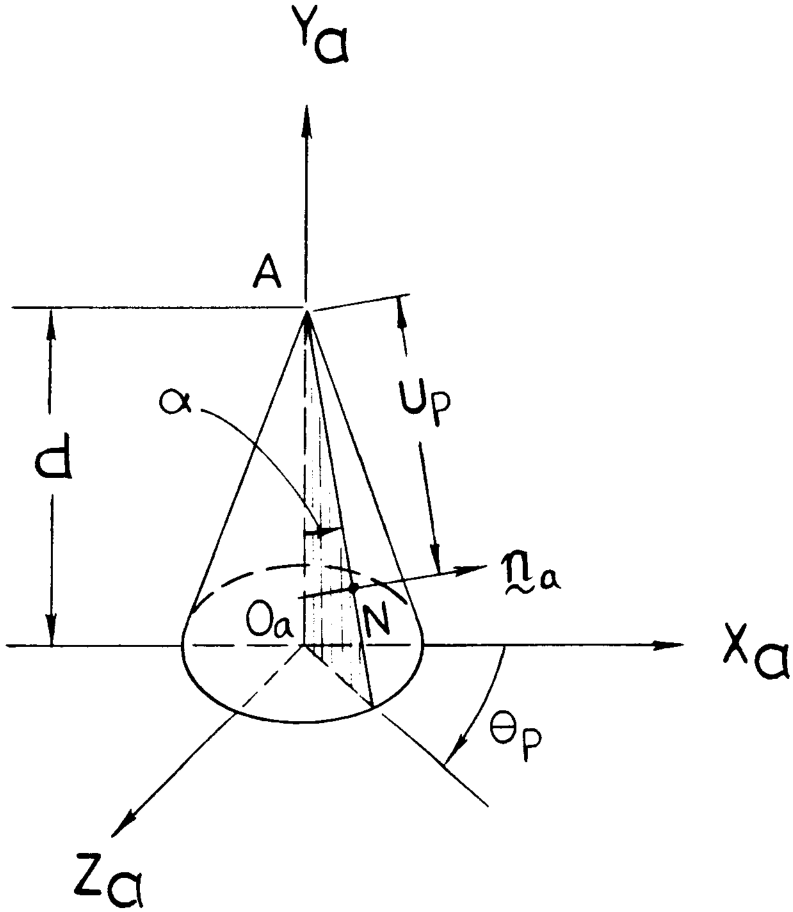


FIG. 4.3.1

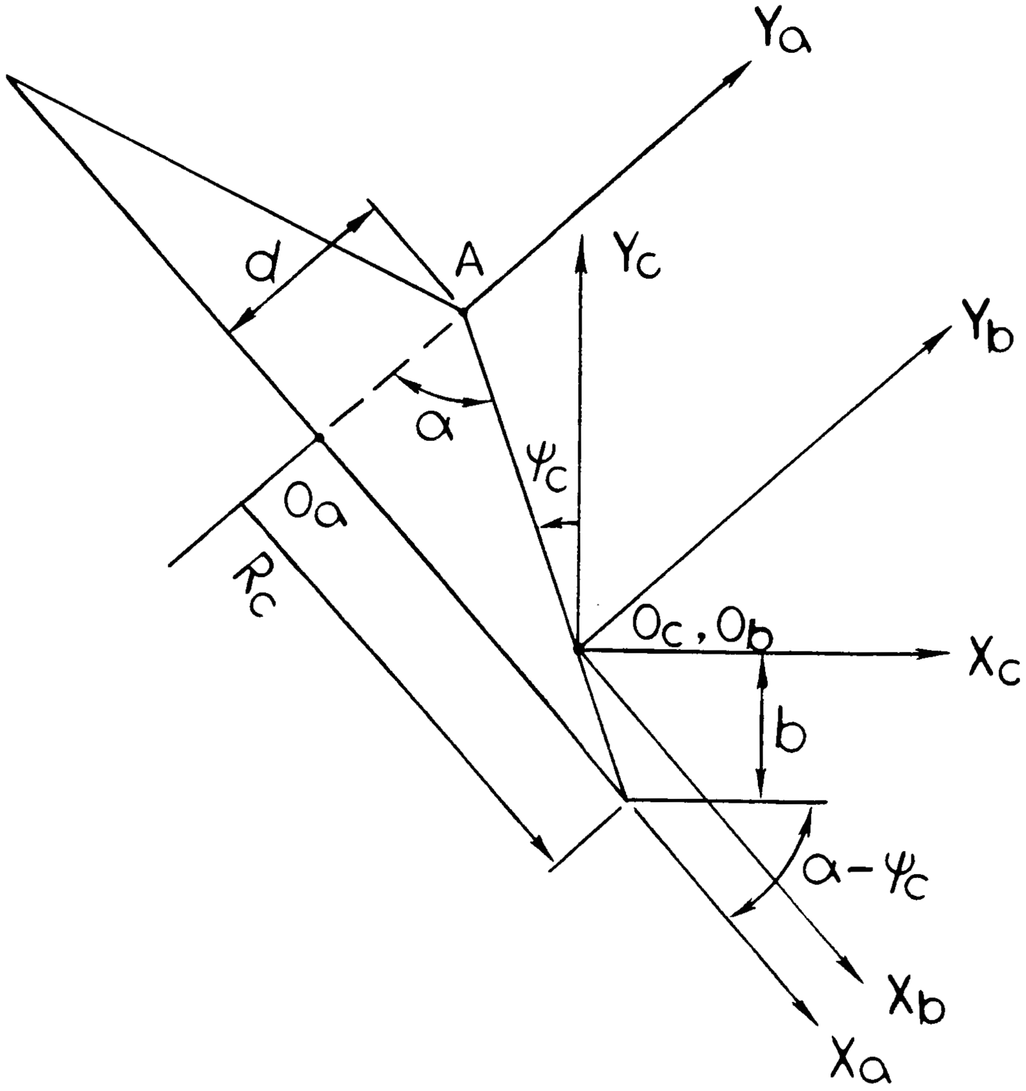


FIG. 4.3.2

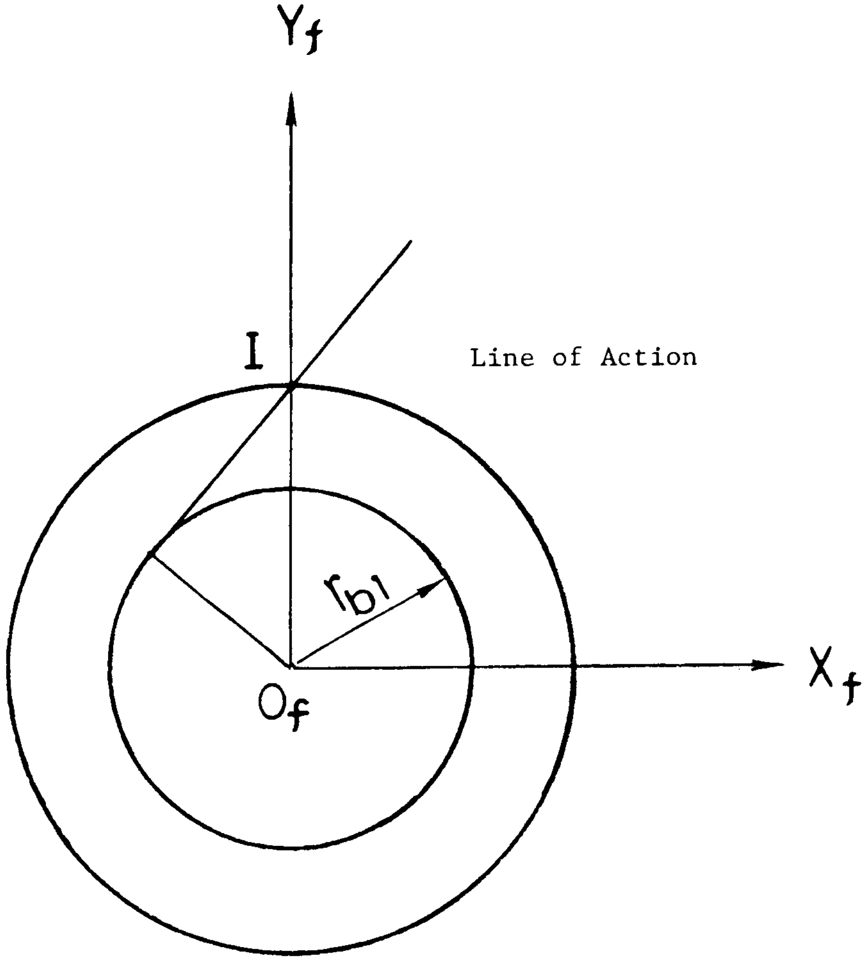
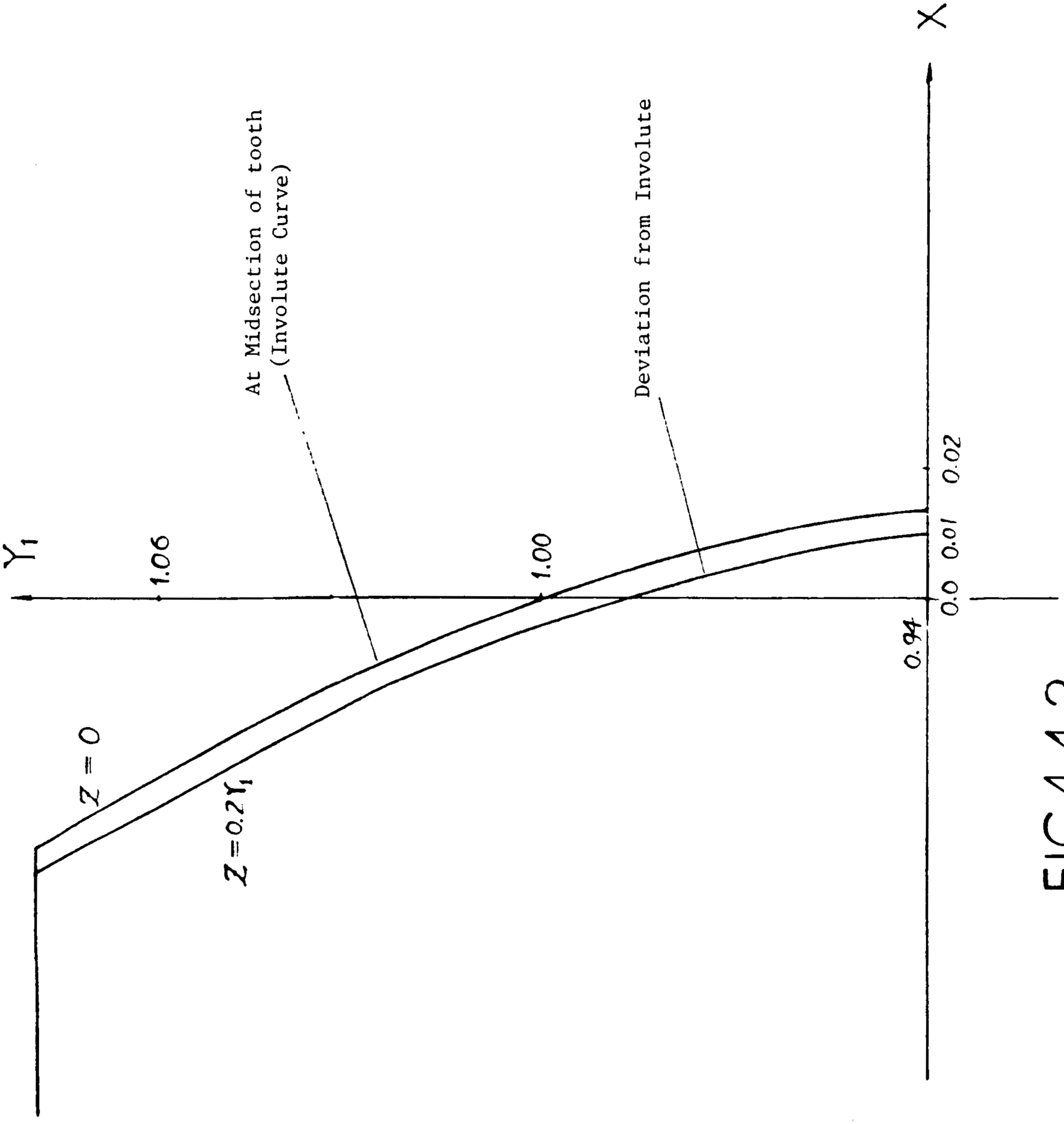


FIG. 4.4.1



Y_1
X_1
1.06
1.00
0.94
0.0
0.01
0.02
At Midsection of tooth
(Involute Curve)
Deviation from Involute
$Z = 0$
$Z = 0.2Y_1$

FIG.4.4.2

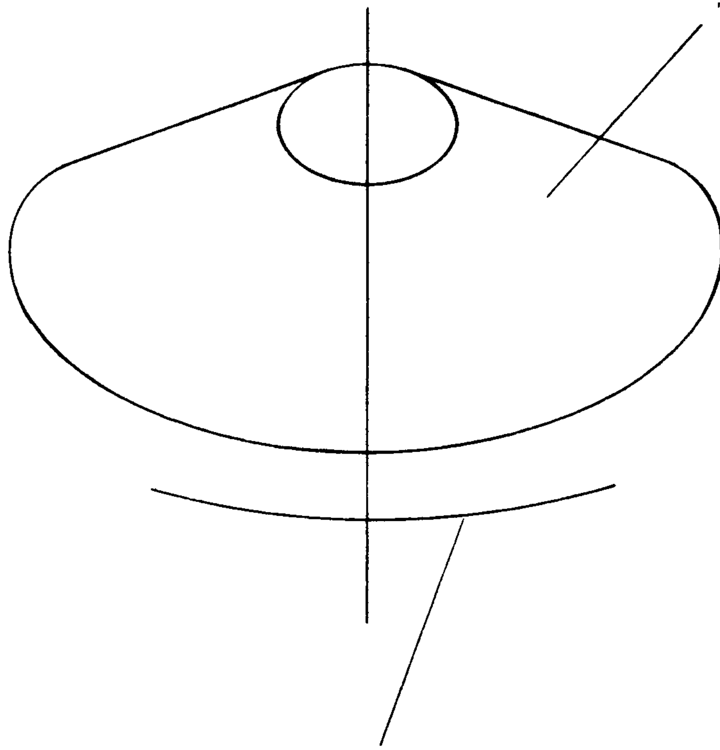


FIG. 4.5.1

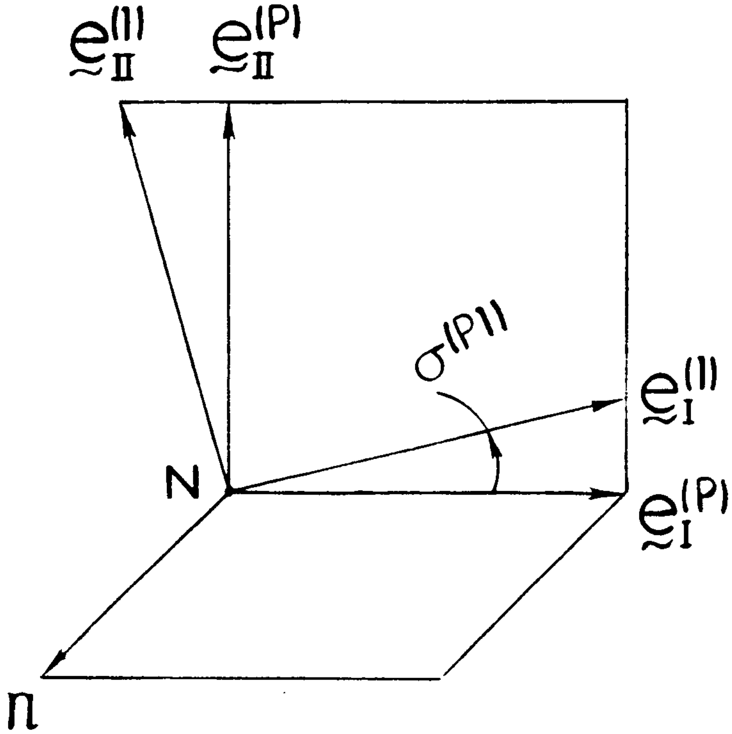


FIG. 4.6.1

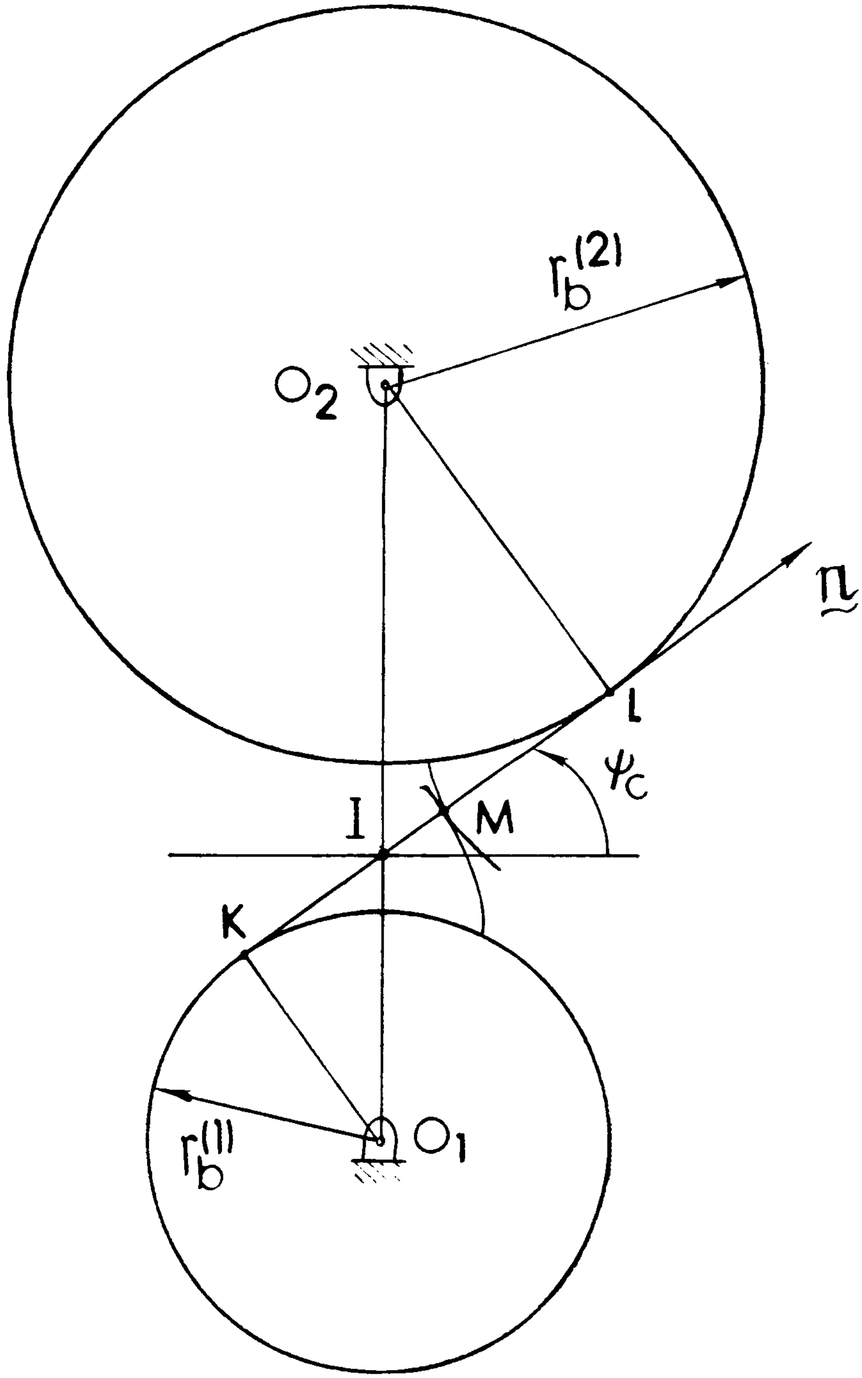


FIG. 4.6.2

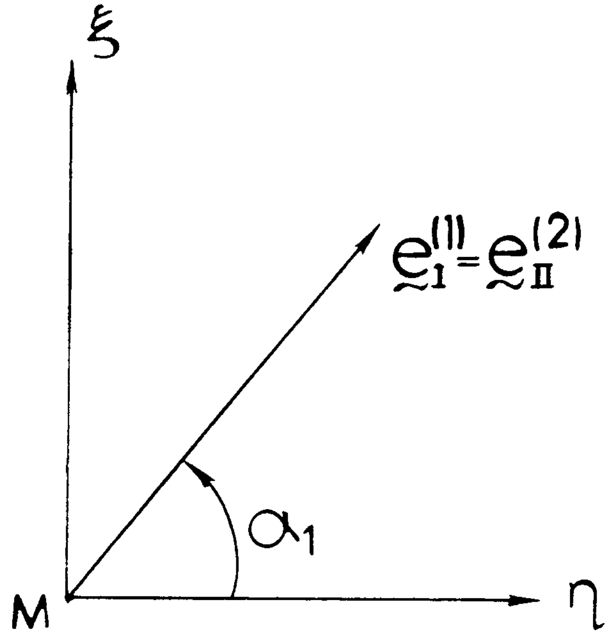

ξ
$\underset{\sim}{e}_{\mathrm{I}}^{(1)} = \underset{\sim}{e}_{\mathrm{II}}^{(2)}$
α_1
M
η

FIG. 4.7.1

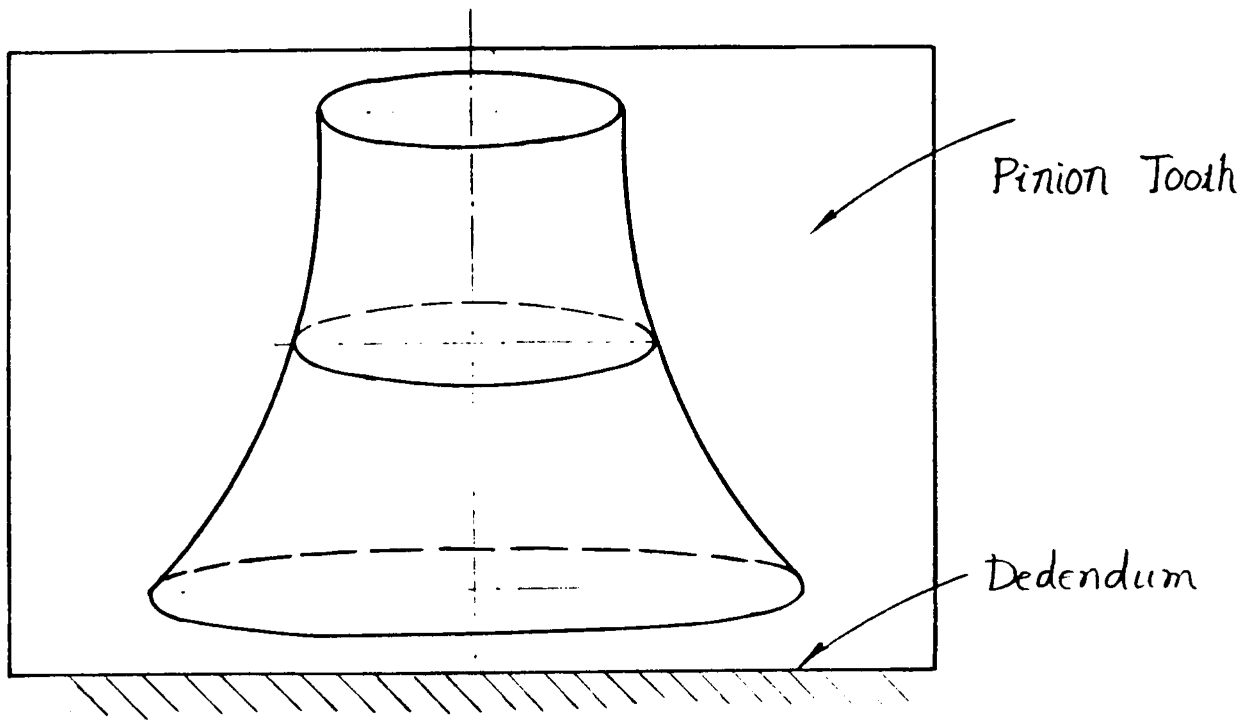
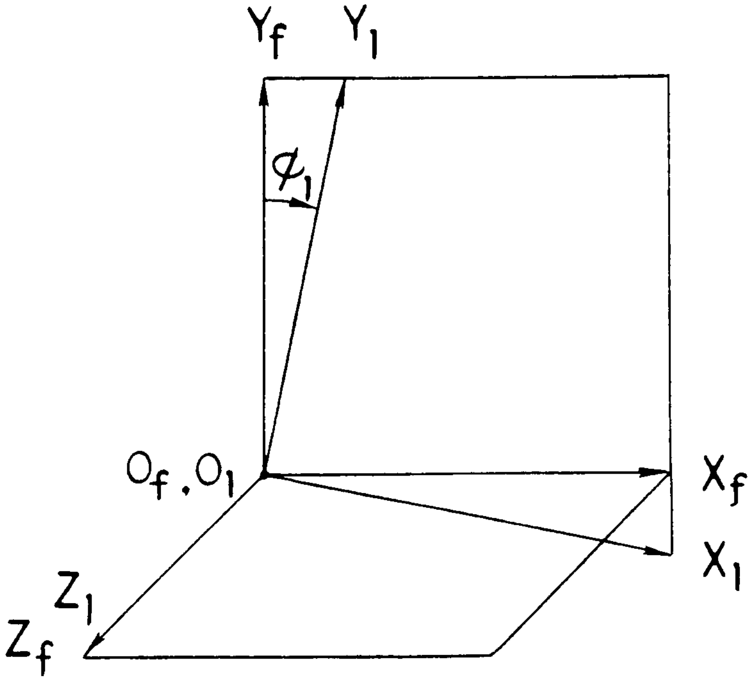


FIG. 4.7.2



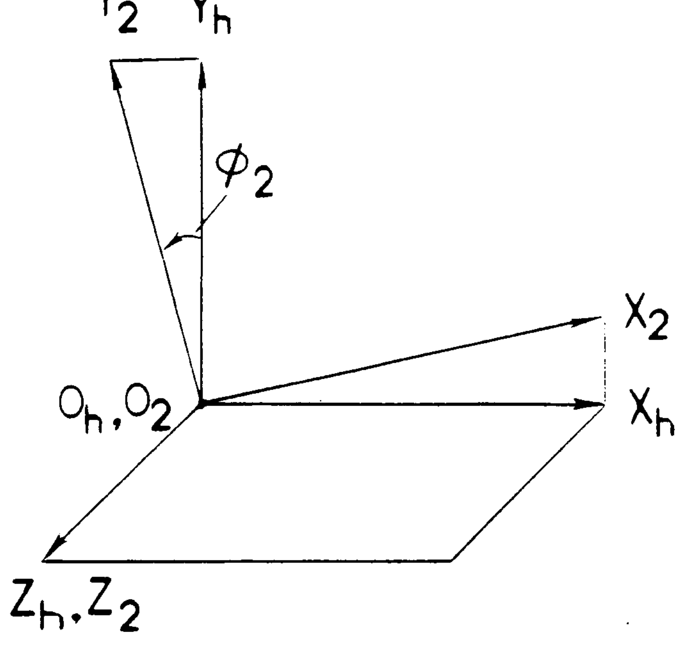


FIG.4.8.1

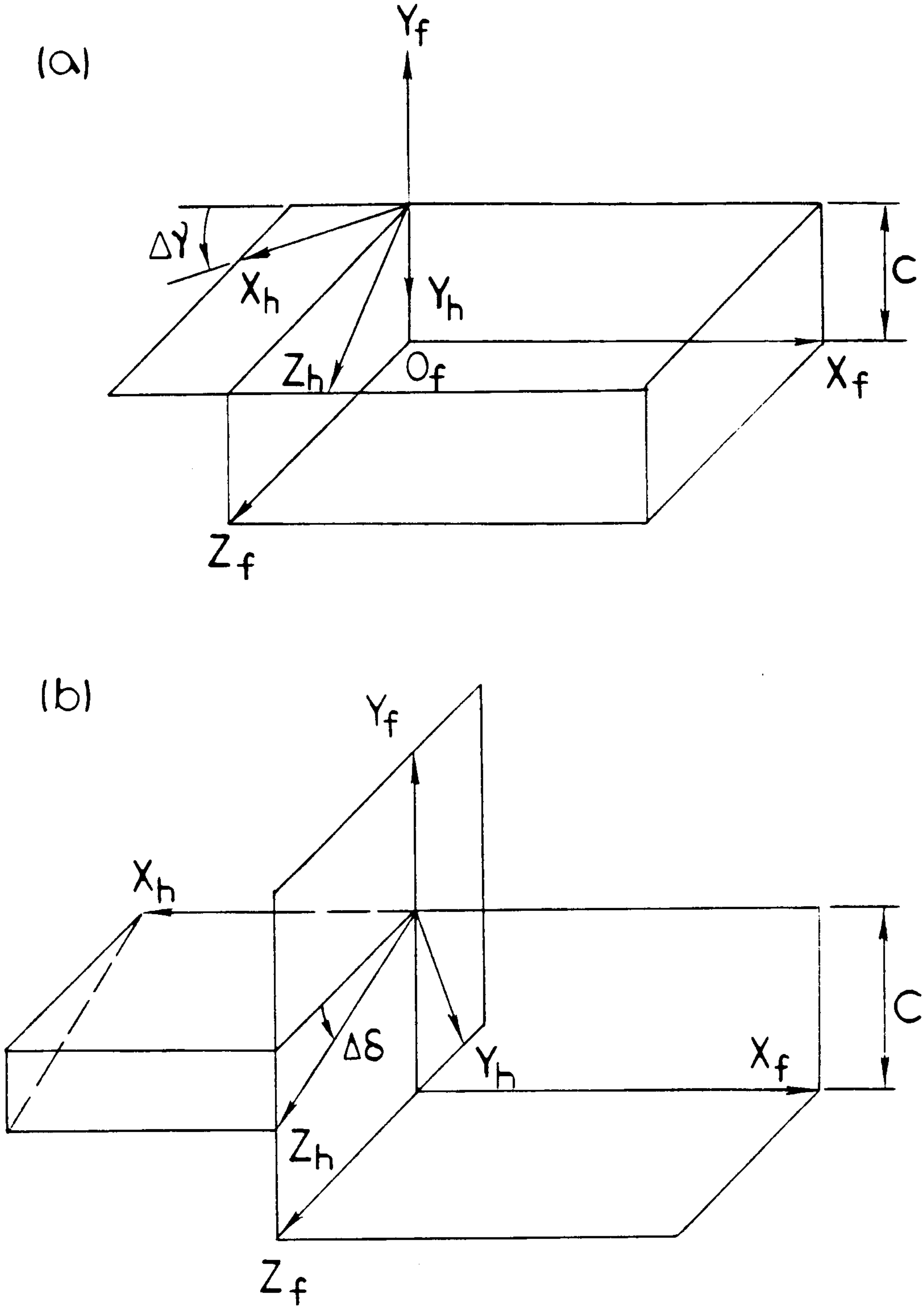

(a)
Y_f
$\Delta\gamma$
X_h
Y_h
C
Z_h
O_f
X_f
Z_f
(b)
Y_f
X_h
$\Delta\delta$
Y_h
X_f
C
Z_h
Z_f

FIG. 4.8.2

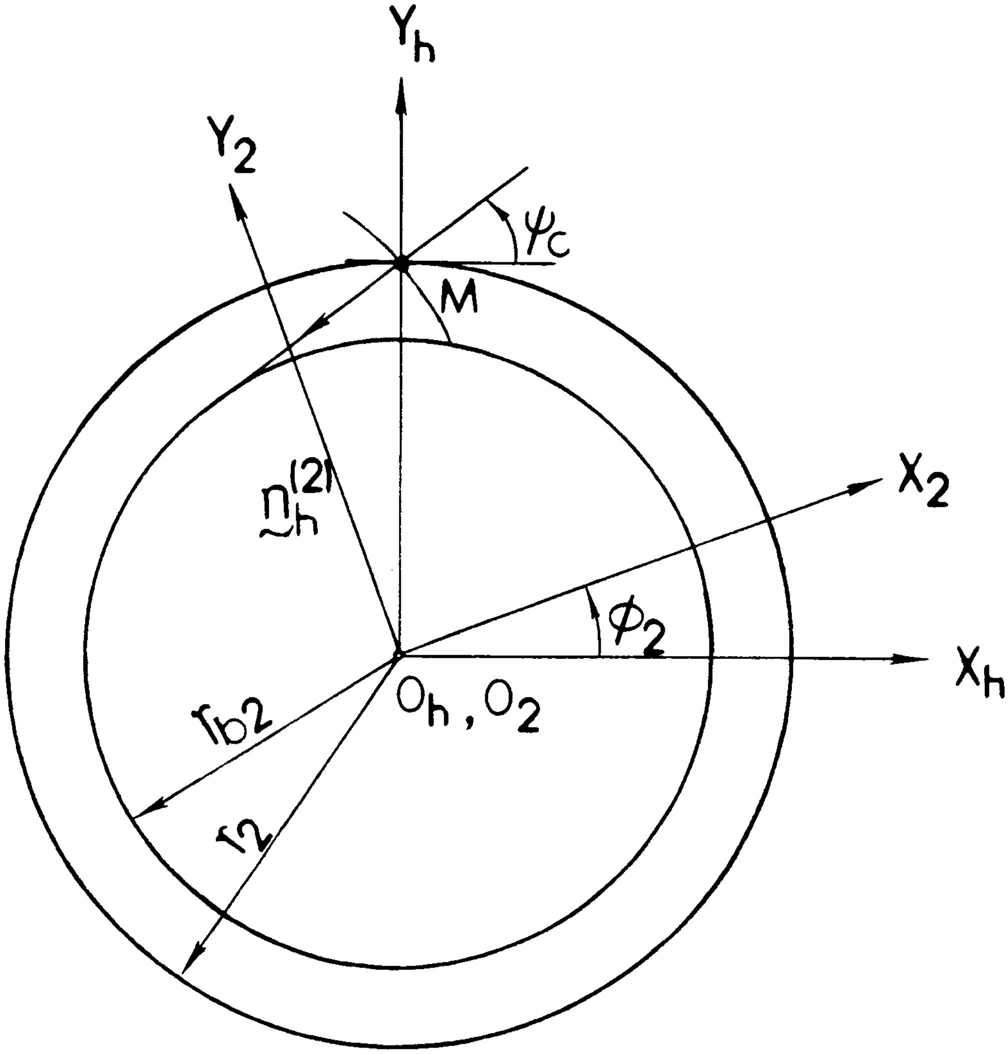


FIG. 4.8.3

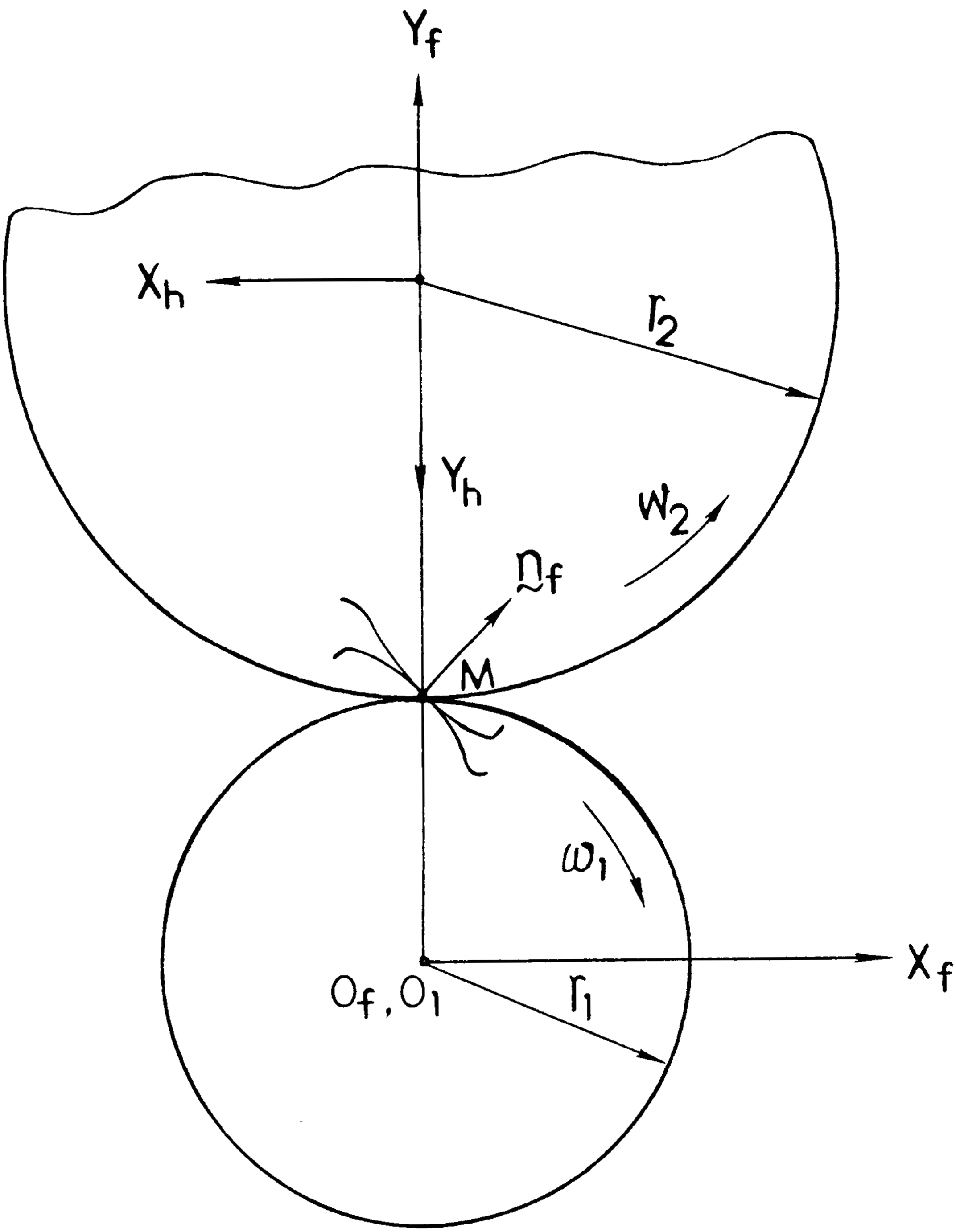


FIG. 4.8.4

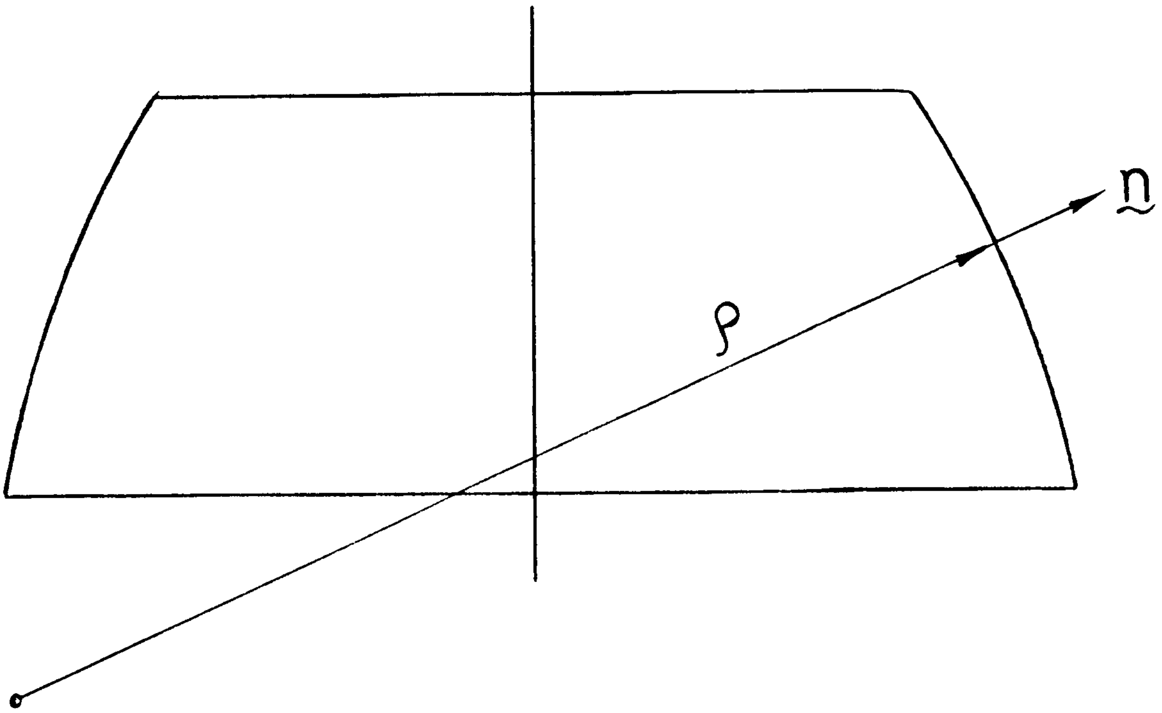


FIG. 4.9.1

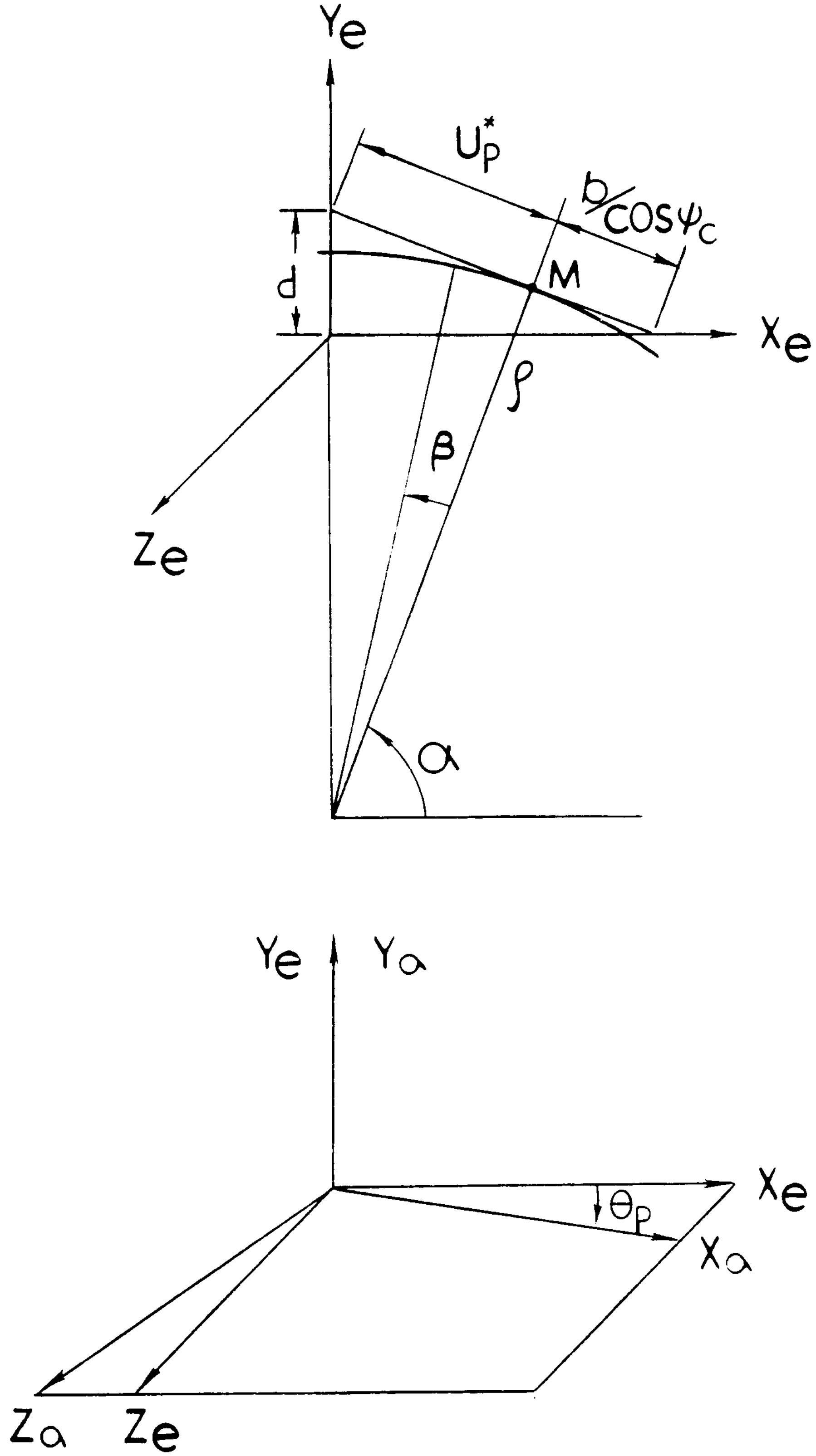


FIG.4.9.2

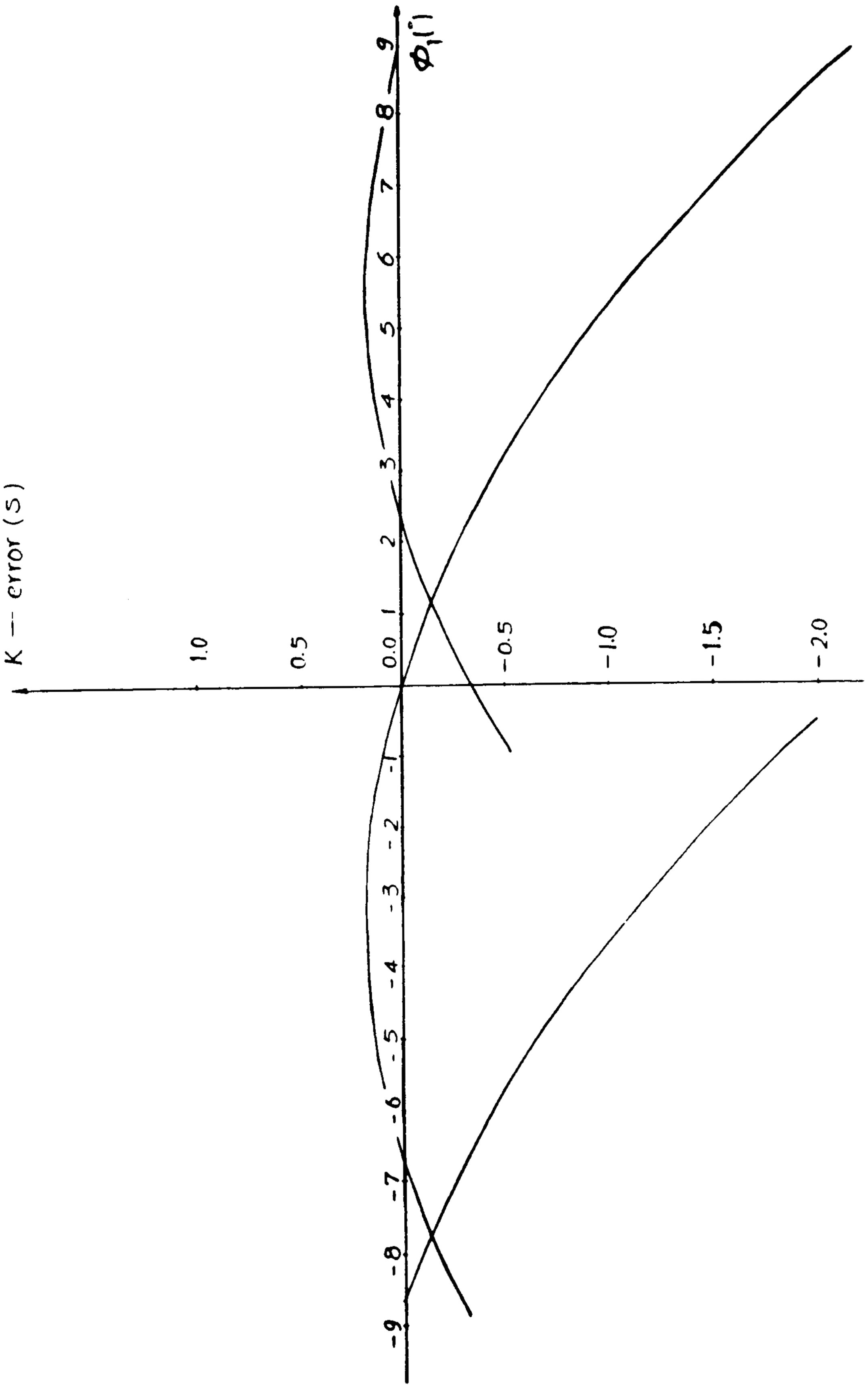


FIG.4.9.3

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Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-4135 AVSCOM-TR-88-C-002		

4. Title and Subtitle	5. Report Date
Spur Gears: Optimal Geometry, Methods for Generation and Tooth Contact Analysis (TCA) Program	April 1988
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Faydor L. Litvin and Jiao Zhang	None (E-3940)
	10. Work Unit No. 1L162209AH76 505-63-51
9. Performing Organization Name and Address University of Illinois at Chicago Circle Department of Mechanical Engineering Chicago, Illinois 60680	11. Contract or Grant No. NAG3-655
12. Sponsoring Agency Name and Address Propulsion Directorate U.S. Army Aviation Research and Technology Activity—AVSCOM Cleveland, Ohio 44135-3191 and NASA Lewis Research Center Cleveland, Ohio 44135-3191	13. Type of Report and Period Covered Contractor Report Final 14. Sponsoring Agency Code

15. Supplementary Notes

Project Manager, Robert F. Handschuh, Propulsion Directorate, U.S. Army Aviation Research and Technology Activity—AVSCOM, Lewis Research Center.

16. Abstract

The contents of this report covers: (i) development of optimal geometry for crowned spur gears; (ii) methods for their generation; and (iii) tooth contact analysis (TCA) computer programs for the analysis of meshing and bearing contact of the crowned spur gears. The developed method for synthesis is used for the determination of the optimal geometry for crowned pinion surface and is directed to reduce the sensitivity of the gears to misalignment, localize the bearing contact, and guarantee the favorable shape and low level of the transmission errors. A new method for the generation of the crowned pinion surface has been proposed. This method is based on application of the tool with a surface of revolution that slightly deviates from a regular cone surface. The tool can be used as a grinding wheel or as a shaver. The crowned pinion surface can be also generated by a generating plane whose motion is provided by an automatic grinding machine controlled by a computer. The TCA program simulates the meshing and bearing contact of the misaligned gears. The transmission errors are also determined.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Spur gears Gears Tooth contact analysis Crowned spur gears	Unclassified – Unlimited Subject Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No of pages	22. Price*
Unclassified	Unclassified	134	A07

NASA FORM 1626 OCT 86

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NASA-Langley, 1988